System Identification – Practical Assignment 8
Instrumental variable methods

Logistics

Please reread the logistics part of lab 2, the same rules will apply to this lab. The only thing that changes is the DropBox link, which will be communicated separately.

Assignment description

In this assignment we will study instrumental variable methods, using *existing datasets* (not the DC motor). Each student is assigned an index number by the lecturer. Then, the student downloads the data file that forms the basis of the assignment from the course webpage. The file contains the identification data in variable `id`, and separately the validation data in variable `val`. From prior knowledge, it is known that the system is of the order given in variable `n` in the data file; and that the disturbance is not white noise, but colored. Thus, for all models we will use $na = nb =$`n`, the value from the datafile.

Your task is to implement the IV algorithm using instruments based on ARX-outputs. To solve the identification problem efficiently in Matlab, it will be useful to rewrite the IV system of equations in a form amenable to matrix left division. To that end, let us take the following form from the lecture slides:

$$\left[\frac{1}{N}\sum_{k=1}^{N} Z(k)\varphi^T(k)\right]\theta = \frac{1}{N}\sum_{k=1}^{N} Z(k)y(k)$$

$$\text{or equivalently: } \tilde{\Phi}\theta = \tilde{Y}$$

where the $(na + nb) \times (na + nb)$ matrix $\tilde{\Phi} = \frac{1}{N}\sum_{k=1}^{N} Z(k)\varphi^T(k)$ and the $(na + nb) \times 1$ vector $\tilde{Y} = \frac{1}{N}\sum_{k=1}^{N} Z(k)y(k)$. Note the tildes, which signify that these quantities are variants of the regressors and of the original system outputs, "modified" by the IVs.

In the equation above, the instrument vector is:

$$Z(k) = [-\hat{y}(k-1), \ldots, -\hat{y}(k-na), u(k-1), \ldots, u(k-nb)]^T$$

where the outputs $\hat{y}$ are **simulated** with the ARX model found earlier. Do not use predicted outputs, as those are correlated with the disturbance and will likely break the IV method!

Recall that $\theta = [a_1, \ldots, a_{na}, b_1, \ldots, b_{nb}]^T$, and that the regressor vector $\varphi(x)$ is the usual one from ARX:

$$\varphi(k) = [-y(k-1), \ldots, -y(k-na), u(k-1), \ldots, u(k-nb)]^T$$

Requirements:

- Identify an ARX model with orders chosen as above and inspect its quality. It is preferable (but not mandatory) to use your own code developed for the ARX lab, as it will give you more directly the simulated outputs that you need to construct the IVs.
- Identify a model with the IV method, with the same orders, using the method described above and ARX-based instruments.

- Compare the quality of the IV model with that of the original ARX model, in simulation.

Optionally, if you still have time, run IV identification also with the simpler instruments:

$$Z(k) = [u(k - nb - 1), \dots u(k - na - nb), u(k - 1), \dots, u(k - nb)]^T$$

and compare the results with the above.

Hints: (i) For simplicity, fill in the vectors Z directly with the $\hat{y}$ and u values, rather than defining polynomials C and D. (ii) Construct $\tilde{\Phi}$, $\tilde{Y}$ efficiently by summing up terms computed using matrix operations in Matlab. (iii) Don't forget to fill in zeros for negative-and-zero time steps in the vectors Z and φ.