Reinforcement learning

Master CPS, Year 2 Semester 1

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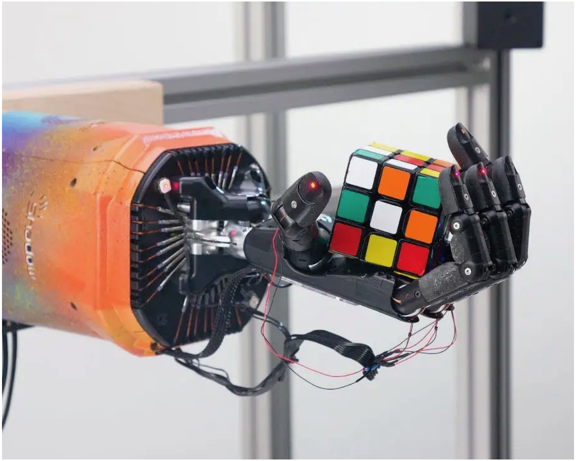
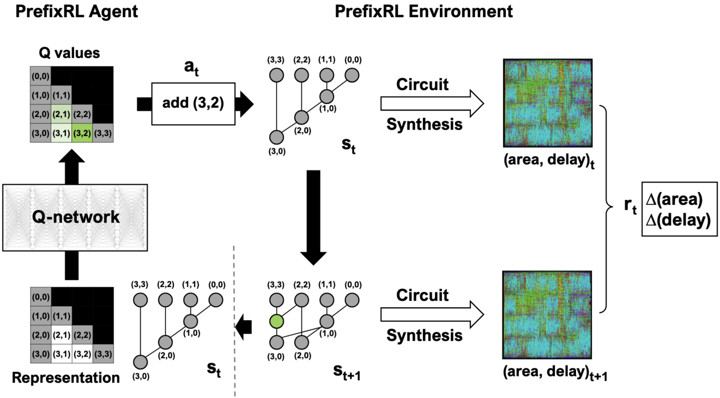
Human-level Atari with DQN (DeepMind)

RL for manipulation of a Rubik's Cube (OpenAI)



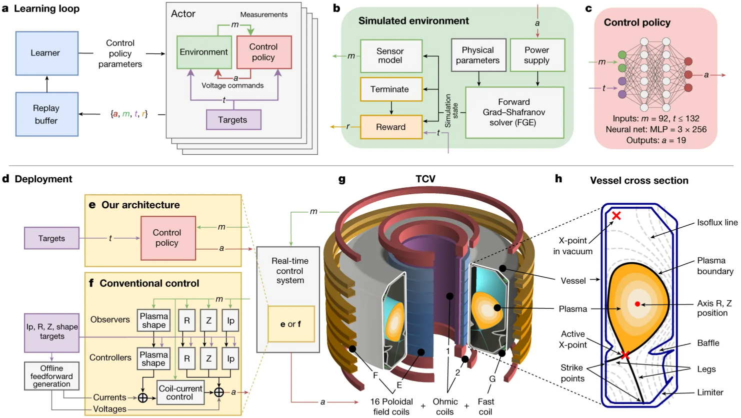
Learn fine control of a large number of actuators, even in the presence of external disturbances.

RL design of digital circuits (Nvidia / Google)



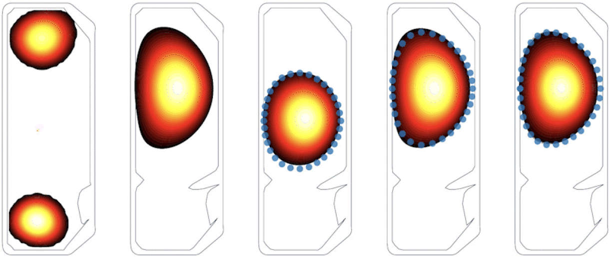
Learn optimal placement of parallel prefix circuits such as adders while optimising for area, delay and power.

RL for plasma control in a fusion reactor (DeepMind)



Learn to control the plasma confinement magnetic field in a simulated fusion reactor. Objective: shape and maintain high-temperature plasma.

RL for plasma control in a fusion reactor (DeepMind)



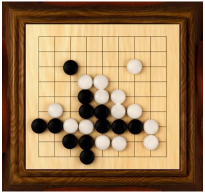
Various plasma shapes obtained by the learned controller, including a novel “droplets” configuration.

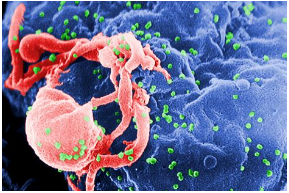
Planning for a domestic robot (UTCluj)

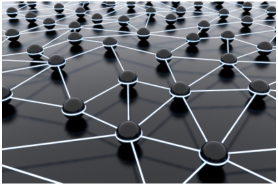
Domestic robot ensures that switches are turned off
High-level control (actions "translated" by low-level controllers into actuator commands)

Other applications

Artificial intelligence, medicine, networks of agents, economics, etc.







Course contents

- **Lecture 1: Reinforcement learning problem**
- Optimal solution
- Exact dynamic programming
- Exact reinforcement learning
- Approximation techniques
- Approximate dynamic programming
- Approximate reinforcement learning

Part I

Reinforcement learning problem

Lecture 1 contents

Why learning?

Learning finds solutions that:

1. cannot be designed in advance
   - the problem is too complex (e.g., control of strongly nonlinear systems)
   - the problem is incompletely known (e.g., robotic exploration of outer space)
2. continuously improve
3. adapt to a changing environment over time

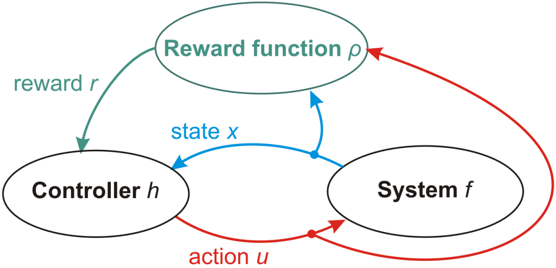
Essential for any **intelligent** system

Model-based methods

We will also focus on **model-based methods**:

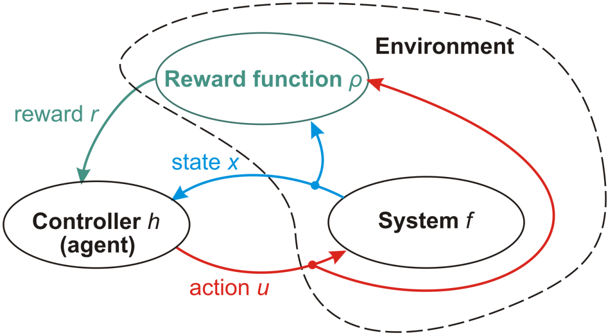
- They form the basis of reinforcement learning (e.g., dynamic programming)
- Useful independently of learning, when model available, as they address complex (e.g., nonlinear) problems

RL principle: control view



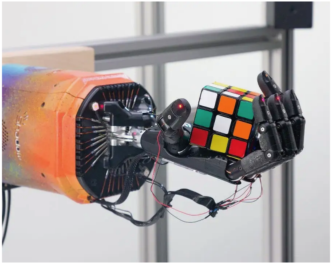
- Interact with system: measure states, apply actions
- Performance feedback in the form of rewards
- Inspired by human and animal learning

RL principle: AI view



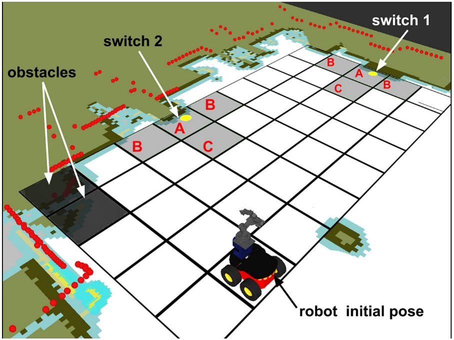
- Agent embedded in an environment
  that receives actions
  and feeds back states and rewards

Example: Rubik's cube manipulation



- States: joint angles, cube and goal positions and orientations
- Actions: 11 bins for each of the 20 actuated joints
- Rewards:
  - distance to the goal state
  - positive reward when a goal is reached
  - negative reward when a cube is dropped

Example: Domestic robot



- States: grid coordinates, switch states
- Actions: move NSEW, toggle switches
- Rewards: when a switch that was on is turned off (and penalty when an off switch is turned on!)

Example of **abstraction**: problem solved high-level, actions implemented by low-level controllers

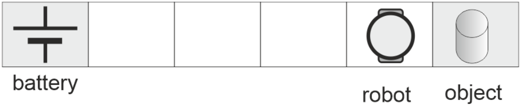
Exact vs. approximate; deterministic vs. stochastic

- **Parts 1–3: exact methods** – discrete states and actions with a small number of values
  - intermediate step, needed to understand the more difficult problem with approximation
  - useful on its own, if the problem can be abstracted into a high-level discrete one
- Parts 4 and onwards: approximate methods – states and actions continuous, or discrete with many values

System can behave:

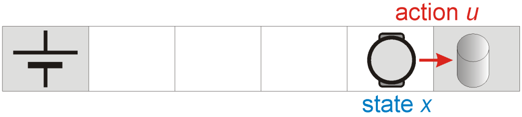
- Deterministically – always responds the same to the same action in the same state
- Stochastically

Simple example: cleaning robot

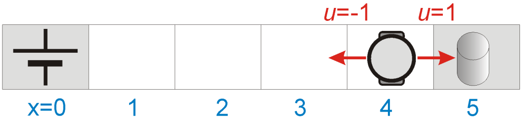


- Cleaning robot in a 1-D world
- Collects trash (reward $+5$) or battery (reward $+1$)
- After either object is collected, episode ends

Cleaning robot: state & action

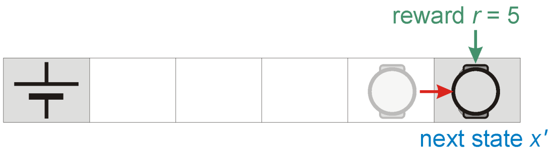


- Robot is in a state x
- and applies an action u (e.g., moves right)



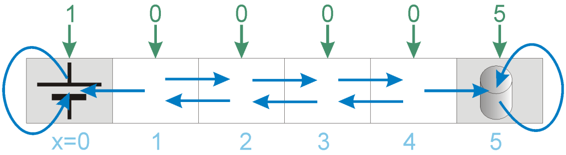
- State space $X = \{0, 1, 2, 3, 4, 5\}$
- Action space $U = \{-1, 1\} = \{\text{left, right}\}$

Cleaning robot: transition & reward



- Robot reaches a new state x'
- and receives a reward r = quality of the transition (here, $+5$ for collecting trash)

Cleaning robot: transition & reward functions



- Transition function (system behavior):

$$x' = f(x, u) = \begin{cases} x & \text{if } x \text{ terminal (0 or 5)} \\ x + u & \text{otherwise} \end{cases}$$

- Reward function (immediate performance):

$$r = \rho(x, u) = \begin{cases} 1 & \text{if } x = 1 \text{ and } u = -1 \text{ (battery)} \\ 5 & \text{if } x = 4 \text{ and } u = 1 \text{ (trash)} \\ 0 & \text{otherwise} \end{cases}$$

- **Note:** Terminal states cannot be exited and are not rewarded!

A note on rewards

- In fact, rewards depend on the **transition** $r = \tilde{\rho}(x, u, x')$
- But x' is determined by (x, u) and can be substituted in the formula:

$$\tilde{\rho}(x, u, x') = \tilde{\rho}(x, u, f(x, u)) = \rho(x, u)$$

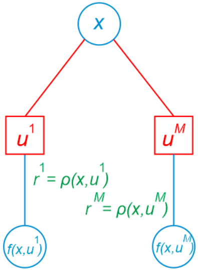
$$r = \rho(x, u) = \begin{cases} 1 & \text{if } x = 1 \text{ and } u = -1 \text{ (battery)} \\ 5 & \text{if } x = 4 \text{ and } u = 1 \text{ (trash)} \\ 0 & \text{otherwise} \end{cases}$$

Deterministic Markov decision proces

Deterministic Markov decision process

Consists of:

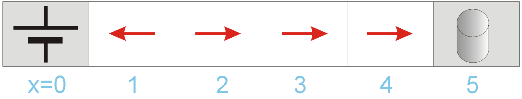
1. State space X
2. Action space U
3. Transition function $x' = f(x, u), \quad f : X \times U \to X$
4. Reward function $r = \rho(x, u), \quad \rho : X \times U \to \mathbb{R}$



1. Introduction
2. Deterministic case
   - Markov decision process
   - Policy and objective
3. Stochastic case
4. Course organization

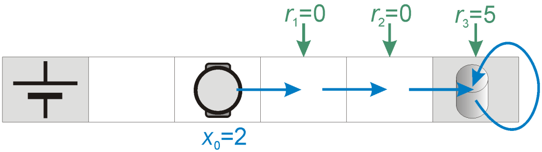
Policy

- Policy h: maps states x to actions u (state feedback)



Example: $h(0) = *$ (terminal state, action irrelevant),
$h(1) = -1$, $h(2) = 1$, $h(3) = 1$, $h(4) = 1$, $h(5) = *$

Cleaning robot: return (value)



Take h that always goes right

$$V^h(2) = \gamma^0 r_1 + \gamma^1 r_2 + \gamma^2 r_3 + \gamma^3 0 + \gamma^4 0 + \dots$$
$$= \gamma^2 \cdot 5$$

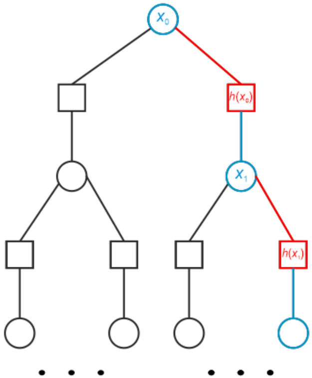
Since x_3 is terminal, all subsequent rewards are 0

General return and objective

Find h that from any x_0 maximizes the discounted return:

$$V^h(x_0) = \sum_{k=0}^{\infty} \gamma^k r_{k+1} = \sum_{k=0}^{\infty} \gamma^k \rho(x_k, h(x_k))$$

Note: There are other types of return!



Discount factor

Discount factor $\gamma \in [0, 1)$:

- induces a "pseudo-horizon" for optimization
- bounds the infinite sum
- represents increasing uncertainty about the future
- helps algorithm convergence

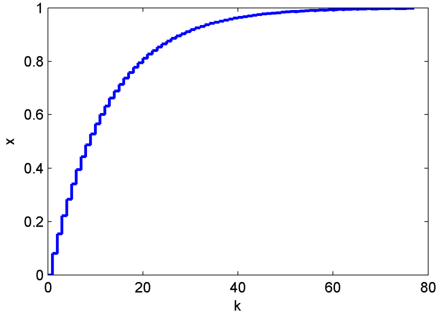
To choose γ, **trade-off** between:

1. Long-term solution quality (large γ)
2. Problem "simplicity" (small γ)

In practice, γ large enough to not ignore important rewards along system trajectories

Example: choosing γ for a first-order linear system

Step response of a first-order linear system:



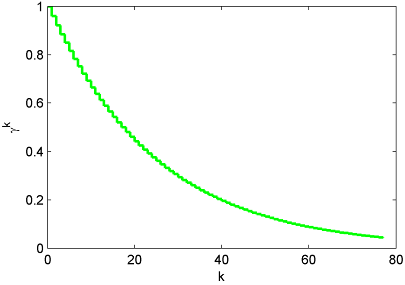
Value of γ so that rewards in steady state are visible from the initial state?

Solution: choosing γ for a first-order linear system

For $k \approx 60$, γ^k should not be too small, e.g.

$$\gamma^{60} \geq 0.05$$
$$\gamma \geq 0.05^{1/60} \approx 0.9513$$

γ^k for $\gamma = 0.96$:



1. Introduction
2. Deterministic case
3. Stochastic case
   - Basics of probabilities
   - RL problem in the stochastic case
4. Course organization

Discrete random variables

- Discrete variable x can take n values, in the set $X = \{x_1, x_2, \dots, x_n\}$.
- Each value is associated with a probability $p(x_1), p(x_2), \dots, p(x_n)$, where $p(x_i) \in [0, 1]$, $\sum_i p(x_i) = 1$. $p : X \to [0, 1]$ is the **probability mass function** (PMF).

Example: The value of a die is a discrete random variable, with $n = 6$ possible values, $x_1 = 1, \dots, x_6 = 6$. For a fair die, $p(x_i) = \frac{1}{6}, \forall i = 1, \dots, 6$

Note: n can grow to infinity; mathematical description remains valid

Expected value (expectation)

- Average of the values, weighted by their probabilities; the value "expected" *a priori*, given the probability distribution:

$$\mathrm{E}\left\{x\right\} = \sum_{x \in X} p(x) x$$

Example: For a fair die, the expectation is

$$\mathrm{E}\left\{x\right\} = \frac{1}{6}1 + \frac{1}{6}2 + \ldots + \frac{1}{6}6 = 7/2$$

- A **function** with a random variable as an argument, $g : X \to \mathbb{R}$ is itself a random variable, with expectation:

$$\mathrm{E}\left\{g(x)\right\} = \sum_{x \in X} p(x) g(x)$$

Example: If faces 1-4 win 1$, and faces 5-6 win 10$,

$$\mathrm{E}\left\{x\right\} = \frac{1}{6}1 + \frac{1}{6}1 + \frac{1}{6}1 + \frac{1}{6}1 + \frac{1}{6}10 + \frac{1}{6}10 = 4\$$$

Independence

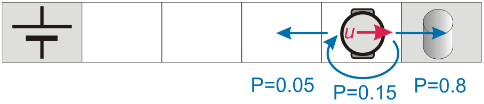
Random variables x, y are independent if the probability of vector $z = (x, y)$ is $p_z(z) = p_x(x) \cdot p_y(y)$, where p_z, p_x, p_y are the PMFs of the three variables. Note: concept extends to any number of variables

Examples:

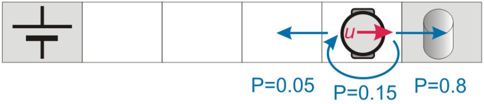
- The values of a die rolled at different times are independent. Among others, the probability of getting a 6 is independent of how many 6s were rolled in previous steps Watch out for gambler's fallacy!
- Temperature values on two consecutive days are not independent! The system is dynamic (has inertia), current values depend on previous ones

Stochastic case

- State no longer evolves deterministically, but **stochastically**
- E.g. cleaning robot "slips" and:
  - moves in the intended direction with probability (w.p.) 0.8
  - stays in place w.p. 0.15
  - moves in the opposite direction w.p. 0.05



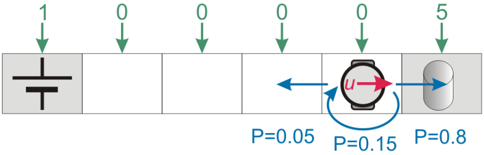
Stochastic cleaning robot: transition function



$\tilde{f}(x, u, x') =$ **probability** of reaching x'
after u has been applied in x

$$\tilde{f}(x, u, x') = \begin{cases} 1 & \text{if } x \text{ terminal and } x' = x \\ 0.8 & \text{if } x \text{ non-terminal, } x' = x + u \\ 0.15 & \text{if } x \text{ non-terminal, } x' = x \\ 0.05 & \text{if } x \text{ non-terminal, } x' = x - u \\ 0 & \text{otherwise} \end{cases}$$

Stochastic cleaning robot: reward function



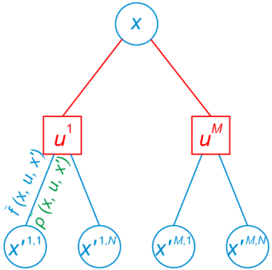
- Transition no longer fully determined by (x, u)
  $\Rightarrow$ the next state x' must be explicitly included
- $\tilde{\rho}(x, u, x')$ = reward on reaching x'
  as a result of action u in x
- For cleaning robot:

$$\tilde{\rho}(x, u, x') = \begin{cases} 5 & \text{if } x \neq 5 \text{ and } x' = 5 \\ 1 & \text{if } x \neq 0 \text{ and } x' = 0 \\ 0 & \text{otherwise} \end{cases}$$

Stochastic Markov decision process

Stochastic Markov decision process

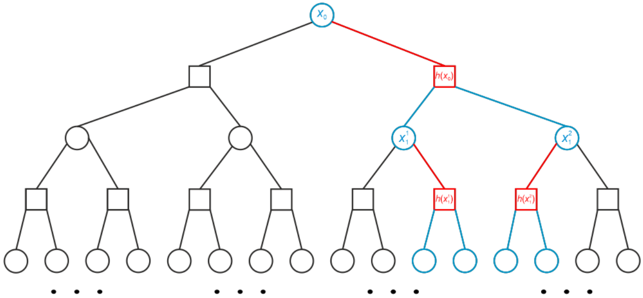
1. State space X
2. Action space U
3. Transition function $\tilde{f}(x, u, x')$, $\quad \tilde{f} : X \times U \times X \to [0, 1]$
4. Reward function $\tilde{\rho}(x, u, x')$, $\quad \tilde{\rho} : X \times U \times X \to \mathbb{R}$



Objective in stochastic case

Find h that from any x_0 maximizes expected discounted return:

$$V^h(x_0) = \mathrm{E}_{x_1, x_2, \dots} \left\{ \sum_{k=0}^{\infty} \gamma^k \tilde{\rho}(x_k, h(x_k), x_{k+1}) \right\}$$

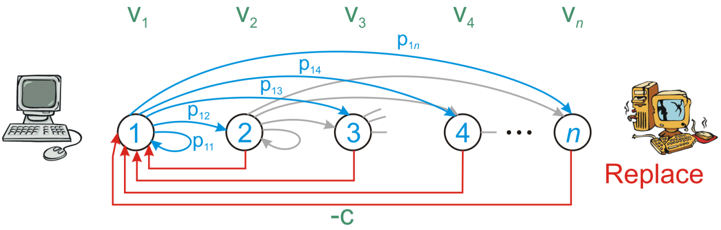


Policy, discount in stochastic case

- Policy $h(x)$ has the same structure,
- discount factor γ has the same meaning

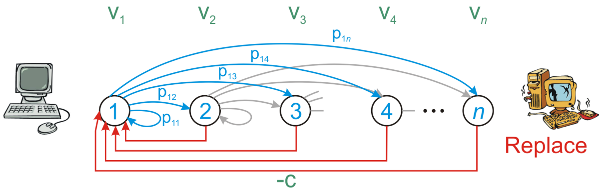
as in the deterministic case

Example: machine replacement



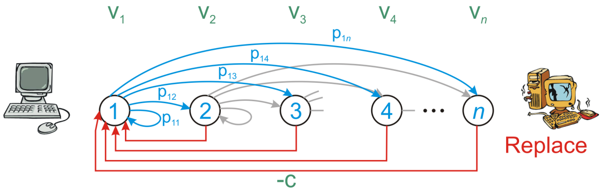
- Machine with *n* different states = wear levels
  1=pristine, *n*=fully degraded
- Produces revenue v_i operating in state i
- Stochastic wear: wear level i transitions to $j > i$ w.p. p_{ij}, remains i w.p. $p_{ii} = 1 - p_{i,i+1} - \dots - p_{i,n}$
- Machine can be instantaneously replaced at any time, paying cost c

Machine replacement: State and action spaces



- State space $X = \{1, 2, \ldots, n\}$
- Action space $U = \{\text{Wait}, \text{Replace}\}$

Machine replacement: Transition and reward functions



- Transition function:

$$\tilde{f}(x=i,u,x'=j)=\begin{cases}p_{ij} & \text{if } u=\text{W and } i\leq j\\ 1 & \text{if } u=\text{R and } j=1\\ 0 & \text{in any other situation}\end{cases}$$

- Reward function:

$$\tilde{\rho}(x=i,u,x'=j)=\begin{cases}v_i & \text{if } u=\text{W}\\ -c+v_1 & \text{if } u=\text{R}\end{cases}$$

Machine replacement: motivation

The RL framework provides a way to formalize and find an optimal decision policy that maximizes the long-term value of the machine

$$V^h(x_0) = \mathrm{E}_{x_1, x_2, \dots} \left\{ \sum_{k=0}^{\infty} \gamma^k \tilde{\rho}(x_k, h(x_k), x_{k+1}) \right\}$$

Key terms in this lecture

- reinforcement learning, RL
- state
- action
- reward
- transition function
- reward function
- Markov decision process
- policy
- return
- discount factor
- random variable
- probability mass function
- expected value

Bibliography

Mandatory material: course slides

Optional books:

- R. Sutton, A. Barto, Reinforcement Learning: An Introduction, ed. 2, 2018.
- D. Bertsekas, Dynamic Programming and Optimal Control, vol. 2, Athena Scientific, 2012.
- D. Bertsekas, Reinforcement Learning and Optimal Control, Athena Scientific, 2024.
- L. Buşoniu, Reinforcement learning and dynamic programming for control, 2012 (lecture notes).

Logistics

Grading:

- 50% labs
- 50% exam
- 10% lecture quizzes

Lab rules:

- labs **mandatory before joining the exam**
- solution = PDF report + code: max 10p if submitted on time, max 5p if late
- solutions must be validated through discussions
- any copied or LLM-generated lab $\Rightarrow$ ineligible and re-enroll

Website, contact

`http://busoniu.net/teaching/rl2025`

Email: `lucian@busoniu.net`, `stefan.pirje@aut.utcluj.ro`

Info

- Course lectures (slides)
- Labs
- Schedule
- etc.

Quiz

Quiz