Switched control strategies in platooning with dwell-time based string stability guarantees

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Abstract— **This paper analyzes the behavior of a platoon of vehicles in the presence of switches between two control modes: Adaptive Cruise Control (ACC), based on local sensing, and Cooperative Adaptive Cruise Control (CACC), which incorporates vehicle-to-vehicle (V2V) communication for improved coordination. Due to the V2V communications, CACC can maintain a shorter inter-vehicular distance, resulting in better performance of the platoon. On the other hand, ACC requires a larger inter-vehicular distance but is robust to degradations introduced by the communication network (time delay, packet loss, etc.). Switching between ACC and CACC can destabilize the platoon if not carefully managed, especially due to the modification of the inter-vehicular distance. To mitigate this, we develop H_∞ specifications for a class of switching systems with lower-bounded state jumps. The proposed methodology requires solving an LMI for the controller design and a minimization problem to compute a lower-bound on the minimum dwell time between two switches. The proposed approach is validated through simulations.**

Autonomous vehicles, traffic control, switched systems

I. Introduction

Given the growth in the number of connected objects, vehicle platooning, which involves an organized set of vehicles moving in coordination with reduced inter-vehicle distances, is a key technology for improving road safety, reducing fuel consumption, and increasing the capacity of existing infrastructures [1], [2]. The stability of such systems needs to be addressed. And one of the main challenges in platooning is to guarantee string stability, an essential property that ensures that disturbances such as variations in speed or acceleration are not amplified as they propagate through the platoon [3].

To achieve this objective, two control modes are often used due to ease of implementation and string stability guarantees: Adaptive Cruise Control (ACC), which relies solely on local measurements such as speed and distance from the vehicle in front [4], and Cooperative Adaptive Cruise Control (CACC), which also uses information exchanged via vehicle-to-vehicle communications (V2V). CACC provides faster response and better coordination but is highly dependent on the availability and reliability of communications [5], [6]. ACC, on the other hand, is less effective in terms of coordination, but does not rely on communications and thus is generally adopted as a backup strategy in case of losing the communication link [7]. Many works in the control theory literature have extensively studied these two modes in terms of string stability and individual stability of the vehicles [4], [8], [9], and achieved performance improvements by applying optimal control or reinforcement learning [10], [11].

However, not many works studied the possibility of switching between the ACC and CACC modes in order to benefit from their advantages while minimizing their inconveniences. Among the few existing works, the switching in [7] is motivated by communication loss whereas [12] is motivated by fuel efficiency. In this context, each vehicle in a platoon can switch dynamically between ACC mode and CACC mode according to pre-established switching rules. However, these transitions raise important issues related to the overall stability of the platoon. For asymptotic stability of switched systems, it is necessary to specify time constraints on the minimum time in a given mode, known as dwell time, to avoid destabilization due to frequent switching [13]. While recent papers, like [14], provide theoretical guarantees of string stability when we switch between the two modes, this is done assuming that both modes maintain the same formation or inter-vehicular distance. In contrast to these works, our main contribution is to provide conditions on the dwell time of each mode so that we have theoretical guarantees of string stability for the resulting switched system *even when the inter-vehicular distance is mode dependent*.

The paper is structured as follows. Section II gives the models used for ACC and CACC. Section III presents instrumental results on a general class of switched systems with state jumps. The main results on platooning are discussed in Section IV. Numerical simulations illustrating the results are provided in Section V. Section VI concludes the paper.

II. Preliminaries and problem formulation

In this section we present a novel analysis for the stability and string stability of a platoon of vehicles when either ACC or CACC is used. The first part is devoted to the model description using ACC or CACC controllers. Secondly, we adapt a result from [15] on H_∞ specifications, to provide the performances of the closed-loop dynamics when one of the aforementioned strategies is implemented.

A. Vehicle Model and Control Modes Description

We consider a platoon of N homogeneous vehicles for which the controller is either ACC or CACC. These two controllers will be detailed later in this section. The goal of each vehicle ν_i, $i = 1, 2, \dots, N$ is to follow the vehicle in front of it while maintaining a desired distance $d_{r,i}$ from it. In this paper, we consider a constant-time-gap policy for each control mode, in which the desired distance at time $t \in \mathbb{R}^+$ is given by

$$d_{r,i}(t) = r + h_{\sigma(t)} v_i(t) \tag{1}$$

where $v_i(t)$ is the velocity of vehicle i at time t, r is the minimum safe distance between two consecutive vehicles, $h_{\sigma(t)}$ is the desired constant time gap for vehicle i and $\sigma : \mathbb{R}^+ \mapsto \{0,1\}$ is the switching rule. Although our results can be easily extended for multiple modes for other switched systems, here we will consider that $\sigma(t)$ can take only two possible values at any time t: 0 for ACC and 1 for CACC.

We assume that all the vehicles in the platoon are identical and have an inertia parameter τ as well as a length L. Let $q_i(t)$ be the position of vehicle i at time t. Let also $d_i(t)$ be the difference between $q_{i-1}(t)$ and $q_i(t)$, from which we subtract L. We define the spacing error $e_i(t)$ as the difference between the desired distance $d_{r,i}(t)$ and the real distance $d_i(t)$

$$e_i(t) = q_{i-1}(t) - q_i(t) - h_{\sigma(t)} v_i(t) - r - L \tag{2}$$

The dynamics between the i^{th} vehicle and the $(i-1)^{th}$ are then given by

$$\dot{e_i}(t) = v_{i-1}(t) - v_i(t) - h_{\sigma(t)} a_i(t) \tag{3a}$$

$$\nu_i : \begin{cases} \dot{v_i}(t) = a_i(t) \\ \dot{a_i}(t) = -\frac{1}{\tau} a_i(t) + \frac{1}{\tau} u_i(t) \end{cases} \tag{3b}$$

$$\dot{u_i}(t) = -\frac{1}{h_{\sigma(t)}} u_i(t) + \frac{1}{h_{\sigma(t)}} \chi_i(t) \tag{3c}$$

where $v_i(t)$ and $a_i(t)$ are respectively the speed and acceleration in dynamics ν_i of the vehicle i at time t, and $u_i(t)$ represents the desired acceleration for the vehicle i obtained as a filtered version of the control input $\chi_i(t)$ of the i^{th} vehicle at time t.

As illustrated in Fig. 1, we model this behavior by a first-order form as described in [16]. And as in [17], a first order low-pass filter defined as $H(s) = h_{\sigma(t)} s + 1$ is applied to the control input $\chi_i(t)$ in order to pre-compensate the time-gap before being used in the model. One key advantage of this control configuration is its independence from the chosen time-gap parameter $h_{\sigma(t)}$ for a constant $\sigma(t)$ [16]. The cancellation of the poles and zeros of both H^{-1} and H in the dynamics of the closed-loop system guarantees that the asymptotic stability properties of the control loop for each vehicle do not rely on the specific value of $h_{\sigma(t)}$. This flexibility allows drivers or system operators to set different time-gaps according to their preferences or operational requirements without compromising stability.

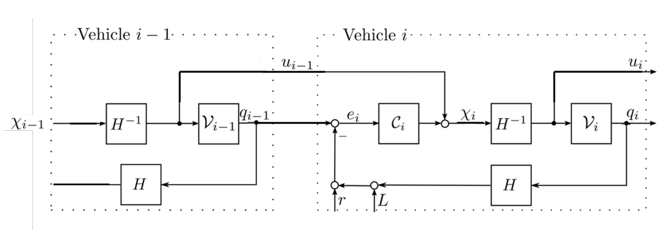

Fig. 1: Schematic representation of ACC/CACC setup

The last element to be specified is the control law χ_i applied to each vehicle. Depending on the control strategy, it can take two different forms. For ACC control, the input law comes from a feedback controller C_i. To regulate e_i we use a PD controller with the controller gains $k_p > 0$ and $k_d > 0$ to be specified. For CACC control, the input law comes from the same feedback controller as ACC, to which is added a feedforward component that incorporates the desired acceleration $\hat{u}_{i-1}(t)$ of the previous vehicle. Note that $\hat{u}_{i-1}(t)$ can be received thanks to the Dedicated Short Range Communication (DSRC) system. In this paper, we consider that the network is perfect and has no communication constraints such as delays, perturbations, packet losses, etc., so $\hat{u}_{i-1}(t) = u_{i-1}(t)$. For previous literature on such effects see [9]. The formal description of the CACC control law is given as follows

$$\chi_i(t) = k_p e_i(t) + k_d \dot{e_i}(t) + \delta_{\sigma(t)} u_{i-1}(t) \tag{4}$$

where the Kronecker coefficient $\delta_{\sigma(t)} = \sigma(t)$ models whether or not there is communication between vehicles.

B. Closed-loop performance with ACC/CACC

The platoon can be seen as a cascade interconnection of systems describing the spacing errors $e_i(t)$ between consecutive vehicles. The dynamics of system i is described by

$$\begin{cases} \dot{x}_i(t) = A_\sigma x_i(t) + B_\sigma \chi_{i-1}(t) \\ \chi_i(t) = C_\sigma x_i(t) \\ x_i(0) = x_i^0 \end{cases} \tag{5}$$

with

$$\begin{aligned} A_\sigma &= \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -\frac{1}{\tau} & \frac{1}{\tau} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{h_\sigma} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -h_\sigma & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau} & \frac{1}{\tau} \\ \frac{k_d}{h_\sigma} & 0 & \frac{\delta_\sigma}{h_\sigma} & \frac{k_p}{h_\sigma} & -k_d & -\frac{1}{h_\sigma} \end{bmatrix}, \\ B_\sigma &= \begin{bmatrix} 0 & 0 & \frac{1}{h_\sigma} & 0 & 0 & 0 \end{bmatrix}^\top, \\ C_\sigma &= \begin{bmatrix} k_d & 0 & \delta_\sigma & k_p & -h_\sigma k_d & 0 \end{bmatrix}, \end{aligned} \tag{6}$$

and state vector $x_i = (v_{i-1} - v_i, a_{i-1}, u_{i-1}, e_i, a_i, u_i)^\top$, arbitrary initial condition x_i^0, input χ_{i-1} and output χ_i. The matrices A_σ, B_σ and C_σ depend on the control mode $\sigma \in \{0,1\}$ of vehicle i and the time gap h_σ.

1) Individual stability: The individual stability of each system can be ensured in both ACC and CACC by a correct design of the PD controller's gains. By using the Routh criterion (see [18]) one can prove that it is sufficient to choose $k_d > \tau k_p$ in order to ensure that $x_i = 0$ is Globally Uniformly Asymptotically Stable (GUAS) for either choice $\sigma \in \{0,1\}$ in (5).

2) String stability: String stability means that the perturbations of one vehicle are not amplified when they propagate in the platoon. Note that the string stability of the platoon without switches, or in the presence of switches that preserve the inter-vehicular distance can be ensured using the $\mathcal{H}_\infty$ norm. We adapt below a theorem from [15] for a system with continuous input and output, respectively χ_{i-1} and χ_i.

The H_∞ condition for the dynamics (5) is described by

$$\frac{\|\chi_i\|_{L_2}}{\|\chi_{i-1}\|_{L_2}} \le \gamma \Leftrightarrow \sup_{\chi_{i-1}\in L_2} \int_0^\infty \chi_i(t)^2 - \gamma^2 \chi_{i-1}(t)^2 \, dt \le 0$$

with $\gamma \in (0,1)$. (7)

Theorem 1: Assume that there exists a positive definite matrix P_σ and a real $\gamma \in (0,1)$ such that

$$A_\sigma^\top P_\sigma + P_\sigma A_\sigma + C_\sigma^\top C_\sigma + \frac{1}{\gamma^2} P_\sigma B_\sigma B_\sigma^\top P_\sigma \preceq 0 \quad (8)$$

The equilibrium solution $x = 0$ of system (5) satisfies

$$\sup_{\chi_{i-1}\in L_2} \left(\int_0^\infty \chi_i(t)^2 - \gamma^2 \chi_{i-1}(t)^2 dt \right) \le 0$$

Proof: A sketch of the proof is as follows. Consider the following Lyapunov function

$$V_\sigma(x_i(t)) = x_i(t)^\top P_{\sigma(t)} x_i(t)$$

where P_σ is positive definite for $\sigma \in \{0,1\}$.
Using (8) one gets that the derivative of V_σ is bounded

$$\begin{aligned}
\dot V_\sigma =& x_i^\top (A_\sigma^\top P_\sigma + P_\sigma A_\sigma) x_i + 2x_i^\top (P_\sigma B_\sigma)\chi_{i-1} \\
\le& - x_i^\top (C_\sigma^\top C_\sigma) x_i - \frac{1}{\gamma^2} x_i^\top (P_\sigma B_\sigma B_\sigma^\top P_\sigma) x_i \\
&+ 2x_i^\top (P_\sigma B_\sigma)\chi_{i-1} \\
\le& - \chi_i^\top \chi_i + \gamma^2 \chi_{i-1}^\top \chi_{i-1} \\
&- \gamma^2 (\chi_{i-1} - \frac{1}{\gamma^2} B_\sigma^\top P_\sigma x_i)^\top (\chi_{i-1} - \frac{1}{\gamma^2} B_\sigma^\top P_\sigma x_i) \\
\le& - \chi_i^\top \chi_i + \gamma^2 \chi_{i-1}^\top \chi_{i-1}
\end{aligned}$$

Integrating from 0 to $+\infty$ and recalling that $V_\sigma(x_i^0) = \lim_{t\to\infty} V_\sigma(x_i(t)) = 0$ one has

$$\begin{aligned}
\int_0^\infty \chi_i(t)^2 - \gamma^2 \chi_{i-1}(t)^2 dt &\le V_\sigma(x_i^0) - \lim_{t\to+\infty} V_\sigma(x_i(t)) \\
&\le {x_i^0}^\top P_{\sigma(0)} x_i^0 = 0.
\end{aligned}$$

■

In other words, both individual and string stability can be guaranteed by a correct controller design, either in an ACC or CACC setup. However, it is well known that in switched systems or hybrid systems, stability of each mode does not necessarily imply stability of the overall system.

3) Objective: Our main objective in this paper is *to analyze the closed-loop in the presence of switching between ACC and CACC*.

ACC imposes a large spacing between vehicles, while CACC requires accurate communication of the control signal of the vehicle ahead. To overcome the inconveniences of both strategies, we propose to switch between the two. This is meant to reduce the spacing between vehicles when the communications can be ensured and preserve the platoon (string) stability when the communications are not reliable (large delay, high packets loss, etc.). The switching can be seen as a source of disturbances in the platoon.

Before giving the main results on the platoon control with a switching strategy, let us provide a new instrumental result that can be stated independently of the application considered in this work.

III. Instrumental result on switched systems with state jumps

In this section, we focus on the analysis of a general class of linear switching systems undergoing jumps in the state at the switching instants. The class of systems considered is not covered by the results in [19], [20]. Let us consider the system can evolve according to $N+1$ modes. We also consider the switching signal $\sigma : \mathbb{R}^+ \mapsto \{0,...,N\}$ and the divergent sequence of switching instants $\{t_k\}_{k\ge 0}$. With this notation at hand, we describe the dynamics under consideration as

$$\begin{cases}
\dot x(t) = A_{\sigma(t)} x(t) \\
x(t_k^+) = \mathcal{J}_{\sigma(t_k^-),\sigma(t_k^+)} x(t_k^-) + \mathcal{C}_{\sigma(t_k^-),\sigma(t_k^+)} \\
x(0) = x_0
\end{cases} \quad (9)$$

where $x(t) \in \mathbb{R}^n$ is the state of the system, x_0 is the initial condition, and $A_i \in \mathbb{R}^{n\times n}$, $i \in \{0,...,N\}$, is the state matrix in the mode i.

In the sequel, we assume that $A_i \in \mathbb{R}^{n\times n}$ is Hurwitz for all $i \in \{0,...,N\}$. At any switching instant t_k, $k \ge 0$ the state undergoes a jump described by the multiplication with the jump matrix $J_{\sigma(t_k^-),\sigma(t_k^+)} \in \mathbb{R}^{n\times n}$ and the sum with a constant vector $\mathcal{C}_{\sigma(t_k^-),\sigma(t_k^+)} \in \mathbb{R}^n$. We emphasize that in the absence of state jumps, a standard approach to ensure the GUAS of the origin for a switching system is based on the computation of a minimal dwell-time between switches [21], i.e., $\exists\, T > 0$ such that $t_{k+1} - t_k \ge T, \forall k \ge 0$. We also note that a dwell-time condition can ensure the origin is GUAS even when the state of the system undergoes jumps that are proportional to the state, i.e., $x(t_k^+) = \mathcal{J}_{\sigma(t_k^-),\sigma(t_k^+)} x(t_k^-)$, see for instance [20]. Nevertheless, for the dynamics (9) the stability of the origin is hampered by the presence of the vectors $\mathcal{C}_{\sigma(t_k^-),\sigma(t_k^+)}$ in the description of the jumps. Thus, we must provide a novel result, stated in Theorem 2 below. In the sequel, for any $\epsilon > 0$ we use the following notation

$$\beta_\epsilon := \max_{i,j\in\{0,...,N\}} \|\mathcal{J}_{i,j}\|_2 \cdot \epsilon + \max_{i,j\in\{0,...,N\}} \|\mathcal{C}_{i,j}\|_2 \quad (10)$$

with $\|.\|_2$ the operator 2-norm.

Definition 1 (adapted from [22]): An open set $O \in \mathbb{R}^n$ is Strongly Globally Recurrent (SGR) for dynamics (9) if for any $x_0 \in \mathbb{R}^n$ there exists $t > 0$ such that $x(t) \in O$.

Remark 1: If O is SGR for dynamics (9) then the trajectory of (9) will recurrently (infinitely many times) enter the set O.

Theorem 2: Suppose that there exist $T, \alpha, \epsilon > 0$ and a collection of positive definite matrices $\{X_0, ..., X_N \in \mathbb{R}^{n\times n}\}$ such that

$$A_i^\top X_i + X_i A_i \prec 0, \quad \forall i \in \{0,...,N\} \quad (11)$$

and

$$\begin{aligned}
&(1+\alpha)\mathcal{J}_{i,j}^\top X_j \mathcal{J}_{i,j} + \frac{1+\alpha}{\alpha\epsilon^2} \mathcal{C}_{i,j}^\top X_j \mathcal{C}_{i,j} I_n \quad (12) \\
&\quad -(e^{-A_i^\top T} X_i e^{-A_i T}) \prec 0, \qquad \forall i \ne j \in \{0,...,N\}.
\end{aligned}$$

As long as $t_{k+1} - t_k > T,\ \forall k \geq 0$ one has that the set $S := \{x \in \mathbb{R}^n \mid \max_{i \in \{0,\dots,N\}} x^\top X_i x \leq \beta_\epsilon^2\}$ is GUAS and the set $\|x\| < \epsilon$ is SGR for the dynamics (9).

Proof: Define the following switching Lyapunov function $V_{\sigma(t)}(x(t)) = x(t)^\top X_i x(t), \forall i \in \{0, ..., N\}, \sigma(t) = i$. We show that $V_{\sigma(t)}$ is decreasing when $\|x(t)\| \geq \epsilon$. At the switching time, the state may jump outside $\|x(t)\| < \epsilon$ but, from the definition of β_ϵ, it remains inside S.
First, we note that (11) ensures that $V_{\sigma(t)}$ is strictly decreasing along the trajectory as far as the system (9) stays in a given mode ($\sigma(t) = i \in \{0, ..., N\}$).

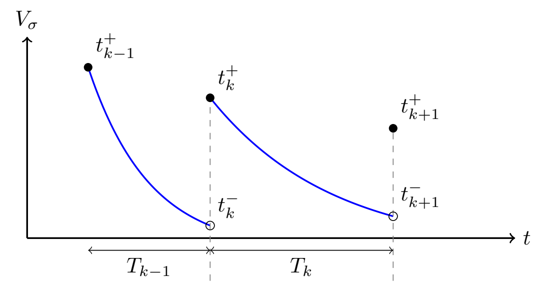

Fig. 2: Representation of the Lyapunov function $V_{\sigma(t)}$

Inequality (12) helps us to ensure the global decrease of $V_{\sigma(t)}$ whatever the number of switches in the dynamics (9), as long as $||x|| \geq \epsilon$.

Indeed, note that $||x|| \geq \epsilon$ implies $\epsilon^2 \leq ||x||^2 = x^\top x$, yielding the following inequality

$$\frac{1+\alpha}{\alpha}\mathcal{C}_{i,j}^\top X_j \mathcal{C}_{i,j} \leq x(t)^\top \left(\frac{1+\alpha}{\epsilon^2 \alpha}\mathcal{C}_{i,j}^\top X_j \mathcal{C}_{i,j}\right) x(t). \quad (13)$$

Therefore, when the dynamics (9) switch from mode i to mode j at time t_{k+1} one has

$$\begin{aligned}
&V_j(x(t_{k+1}^+)) - V_i(x(t_k^+)) = \\
&x(t_{k+1}^+)^\top X_j x(t_{k+1}^+) - x(t_k^+)^\top X_i x(t_k^+) = \\
&(x(t_{k+1}^-)^\top \mathcal{J}_{i,j}^\top + \mathcal{C}_{i,j}^\top) X_j (\mathcal{J}_{i,j} x(t_{k+1}^-) + \mathcal{C}_{i,j}) \\
&- x(t_{k+1}^-)^\top e^{-A_i^\top (t_{k+1}-t_k)} X_i e^{-A_i (t_{k+1}-t_k)} x(t_{k+1}^-) < \\
&(1+\alpha) x(t_{k+1}^-)^\top \mathcal{J}_{i,j}^\top X_j \mathcal{J}_{i,j} x(t_{k+1}^-) + \frac{1+\alpha}{\alpha}\mathcal{C}_{i,j}^\top X_j \mathcal{C}_{i,j} \\
&- x(t_{k+1}^-)^\top e^{-A_i^\top T} X_i e^{-A_i T} x(t_{k+1}^-)
\end{aligned}$$

When $\|x\| \geq \epsilon$, one obtains that

$$\begin{aligned}
&V_j(x(t_{k+1}^+)) - V_i(x(t_k^+)) < \\
&x(t_{k+1}^-)^\top \Big((1+\alpha)\mathcal{J}_{i,j}^\top X_j \mathcal{J}_{i,j} + \frac{1+\alpha}{\alpha\epsilon^2}\mathcal{C}_{i,j}^\top X_j \mathcal{C}_{i,j} I_n \\
&- e^{-A_i^\top T} X_i e^{-A_i T} \Big) x(t_{k+1}^-)
\end{aligned} \quad (14)$$

Consequently, as far as $\|x\| \geq \epsilon$ the trajectory of (9) converges inside the ball $\|x\| \leq \epsilon$. Moreover, the state of the system is attracted in the set S from which it doesn't escape. ∎

Remark 2: When $\mathcal{C}_{\sigma(t_k^-),\sigma(t_k^+)} = 0$, then (12) in Theorem 2 becomes

$$\mathcal{J}_{i,j}^\top X_j \mathcal{J}_{i,j} - (e^{-A_i^\top T} X_i e^{-A_i T}) \prec 0, \forall i \neq j \in \{0, ..., N\} \quad (15)$$

which aligns with the conditions outlined in [15], to ensure that the origin is a globally asymptotically stable equilibrium of (9).

IV. Performance of the switching closed-loop system

This section provides a dwell-time condition ensuring that any input disturbance introduced by switching is compensated before the next switch and the error remains in a predefined bounded set.

To characterize the string stability of (5) when σ is time varying, we adapt (7) to establish a cost function $J(\sigma)$ such that

$$J(\sigma) := \sup_{\chi_{i-1} \in L_2} \left(\sum_{k=0}^{\infty} \int_{t_k^+}^{t_{k+1}^-} \chi_i(t)^2 - \gamma^2 \chi_{i-1}(t)^2 dt \right) \leq 0 \quad (16)$$

where at each instant t_k for $k \geq 0$ there is a switch in system mode and a discontinuity in the input and/or output, regardless of the sequence of modes σ chosen among $\{\mathrm{ACC}, \mathrm{CACC}\}$.

A. Switching state-space model for platooning

At the price of a certain conservatism, we replace in (5) the input χ_{i-1} with the worst possible input between two consecutive switches. In other words, we replace χ_{i-1} with $L_\sigma x_i$ where L_σ is defined below. This will allow to provide a close-loop version of (5).

Let us introduce the following matrices:

$$\begin{aligned}
H_\sigma &= A_\sigma + B_\sigma L_\sigma \\
Q_\sigma &= C_\sigma^\top C_\sigma - \tfrac{1}{\gamma^2} P_\sigma B_\sigma B_\sigma^\top P_\sigma \\
L_\sigma &= \tfrac{1}{\gamma^2} B_\sigma^\top P_\sigma
\end{aligned} \quad (17)$$

where P_σ is the positive definite and stabilizing solution of the Ricatti equation

$$A_\sigma^\top P_\sigma + P_\sigma A_\sigma + C_\sigma^\top C_\sigma + \frac{1}{\gamma^2} P_\sigma B_\sigma B_\sigma^\top P_\sigma = 0. \quad (18)$$

With the notation in (5), we can rewrite (18) as

$$H_\sigma^\top P_\sigma + P_\sigma H_\sigma + Q_\sigma = 0 \quad (19)$$

and the dynamics (5) leads to a closed-loop of the form

$$\begin{cases} \dot{x}_i(t) = H_{\sigma(t)} x_i(t) \\ x_i(t_k^+) = \mathcal{J} x_i(t_k^-) + \mathcal{C}(t_k) \\ x_i(0) = x_i^0 \end{cases} \quad (20)$$

with the jump matrix $\mathcal{J}$ defined below and the matrix $\mathcal{C}(t_k)$ such that $|\mathcal{C}(t_k)| \leq \mathcal{C}$, a constant matrix independent of t_k also defined below. Note that all the spatial variables (position, velocity and acceleration) are continuous and only the value of the error e_i jumps at the switching instant. So we use equation (2) to get

$$e_i(t_k^+) - e_i(t_k^-) = -(h_{\sigma(t_k^+)} - h_{\sigma(t_k^-)}) v_i(t_k) \quad (21)$$

We therefore have the jump of e_i which can be expressed as a function of v_i, which satisfies $0 \leq v_i \leq v_{\lim}$ (with $v_{\lim} = 40m/s \approx 130km/h$ for example). Since the only two modes associated with $\sigma(t)$ are ACC ($\sigma(t) = 0$) and

CACC ($\sigma(t)=1$), we define $|h_1-h_0|=|h_0-h_1|=|\Delta h|$ which is constant for any switch.s This leads to the equation $e_i(t_k^+)-e_i(t_k^-)\leq|\Delta h|v_{\text{lim}}$, yielding

$$\mathcal{J}=I_d \qquad \mathcal{C}=\begin{bmatrix}0 & 0 & 0 & |\Delta h|v_{\text{lim}} & 0 & 0\end{bmatrix}^\top \tag{22}$$

Note that these matrices are independent of the switching time and of the switching mode.

B. String stability of the switching system

To guarantee string stability as described in (16), we must first introduce the following lemma.

Lemma 1: For the switched linear system (20), the following upper bound holds

$$J(\sigma)\leq\sum_{k=0}^{\infty}x_i(t_k^+)^\top P_{\sigma(t_k^+)}x_i(t_k^+) \tag{23}$$

where P_σ for each $\sigma\in\{0,1\}$ is the stabilizing positive definite solution to the inequality (8).

Proof: Straightforward computation gives

$$\begin{aligned}
J(\sigma) &= \sup_{\chi_{i-1}\in L_2}\left(\sum_{k=0}^{\infty}\int_{t_k^+}^{t_{k+1}^-}\chi_i(t)^2-\gamma^2\chi_{i-1}(t)^2dt\right)\\
&\leq \sum_{k=0}^{\infty}\left(\sup_{\chi_{i-1}\in L_2}\int_{t_k^+}^{t_{k+1}^-}\chi_i(t)^2-\gamma^2\chi_{i-1}(t)^2dt\right)\\
&\leq \sum_{k=0}^{\infty}\left(\sup_{\chi_{i-1}\in L_2}\int_{t_k^+}^{\infty}\chi_i(t)^2-\gamma^2\chi_{i-1}(t)^2dt\right)\\
&\leq \sum_{k=0}^{\infty}\left(x_i(t_k^+)^\top P_{\sigma(t_k^+)}x_i(t_k^+)\right)
\end{aligned} \tag{24}$$

■

By replacing $\mathcal{J}_{ij}$ and $\mathcal{C}_{ij}$ with $\mathcal{J}$ and $\mathcal{C}$ in (10), Theorem 2 leads to the following main result.

Theorem 3: Assume that, for some $T>0$, there exist $\alpha,\epsilon>0$ and a collection of positive definite matrices $\{Z_0,Z_1\}$ of compatible dimensions such that

$$H_\sigma^\top Z_\sigma+Z_\sigma H_\sigma+Q_\sigma\prec 0,\quad\forall\sigma\in\{0,1\} \tag{25}$$

and

$$\begin{aligned}
&(1+\alpha)\mathcal{J}^\top Z_{\bar\sigma}\mathcal{J}+\frac{1+\alpha}{\alpha\epsilon^2}\mathcal{C}^\top Z_{\bar\sigma}\mathcal{C}I_n\\
&\quad-(e^{-H_\sigma^\top T}(Z_\sigma-P_\sigma)e^{-H_\sigma T})\prec 0,\quad\forall\sigma\neq\bar\sigma\in\{0,1\}
\end{aligned} \tag{26}$$

Then, the following hold:

1) The set defined by $S:=\{x\in\mathbb{R}^n\mid\max_{\sigma\in\{0,1\}}x^\top Z_\sigma x\leq\beta_\epsilon^2\}$ is GUAS and the set $\|x_i\|\leq\epsilon$ is GSR for the switched linear system (20).
2) Moreover, any trajectory of the switched linear system with zero initial condition satisfies

$$J(\sigma)\leq 0,\quad\forall\sigma. \tag{27}$$

Proof: 1) Let us introduce $X_\sigma=Z_\sigma-P_\sigma$ with Z_σ a positive definite matrix solving the LMI (25) and P_σ the solution to the Ricatti equation (19). We notice that (11) and (12) rewrite as (25) and (26), respectively. Therefore, Theorem 2 ensures that S is GUAS and $\|x_i\|\leq\epsilon$ is GSR, proving the first statement.

2) Let us define the following change of variable $z_i(t)=x_i(t)^\top Z_{\sigma(t)}x_i(t)$. Using (26) one gets

$$z_i(t_{k+1}^+)-z_i(t_k^+)+x_i(t_k^+)^\top P_{\sigma(t_k^+)}x_i(t_k^+)\leq 0.$$

Summing up for all $k\geq 0$ and remembering that $z_i(t)$ is positive on $\mathbb{R}$ because Z_i is positive definite, we obtain

$$\begin{aligned}
&\sum_{k=0}^{\infty}x_i(t_k^+)^\top P_{\sigma(t_k^+)}x_i(t_k^+)\leq\sum_{k=0}^{\infty}z_i(t_k^+)-\sum_{k=0}^{\infty}z_i(t_{k+1}^+)\\
&\leq z_i(0)-\lim_{k\to+\infty}z_i(t_k^+)\leq {x_i^0}^\top Z_{\sigma(0)}x_i^0.
\end{aligned}$$

When $x_i^0\mapsto 0$, from Lemma 1, we get that (16) holds. ■

Convergence to the origin is impossible due to the jump matrix $\mathcal{C}$, but we can still ensure string stability of the platoon when the conditions in the Theorem are met.

Remark 3: Note that the LMI (25) can be numerically solved in a solver for each σ. Thanks to this, all the matrices in (26) are already known and one can use a minimization algorithm with the constraint (26) to find an α and the minimum dwell time T (for any mode σ).

Remark 4: When ACC and CACC impose the same time gap, (26) in Theorem 3 becomes

$$\mathcal{J}^\top Z_{\bar\sigma}\mathcal{J}-(e^{-H_\sigma^\top T}(Z_\sigma-P_\sigma)e^{-H_\sigma T})\prec 0,\forall\sigma\neq\bar\sigma\in\{0,1\} \tag{28}$$

and the set S reduces to the origin.

A shortcoming of the current approach is that it does not take into account the loss of communications (packet losses or delays) in the CACC mode. This issue has already been well explored for pure CACC systems in [5], [6] etc. when the maximum delay is bounded. Our idea is to switch to ACC when a major failure causes the delay to exceed the maximum allowable delay. Dealing with imperfect communications in the CACC mode can be a future extension.

V. Numerical validation

To evaluate the performances of the proposed switching control, we first consider a platoon of $N=4$ vehicles, then a platoon of $N=20$ vehicles for a more global view. The controller gains are $k_p=6$, $k_d=4$ and $\tau=0.1$. The time gap is $h_0=2$ in ACC mode and $h_1=1$ in CACC mode. The switching signal is chosen to verify the dwell time constraints in Theorem 3. Precisely, we consider that σ alternates between CACC and ACC with $T_{\text{CACC}}=15s>14.4s$ and $T_{\text{ACC}}=30s>25.01s$. Finally, we consider the sinusoidal input signal $\chi_0(t)=\sin(0.2*t)$.

The simulations emphasize two well-known characteristics for platooning without switching control. First, the tracking error without disturbances is guaranteed, so individual stability is respected. Second, in each mode there is a reduction in the propagation of the disturbance in the platoon, so string stability is guaranteed. Moreover, the simulations illustrate our main results, emphasizing that the jumps in the error are upper-bounded and are independent of the number of vehicles in the platoon. Also, as far as the dwell times are

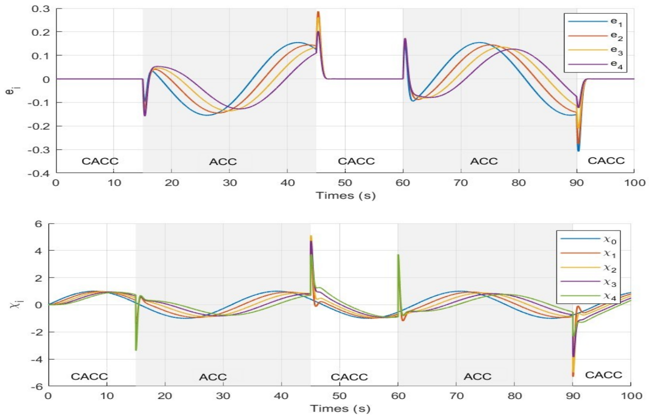

Fig. 3: Platoon dynamic response for $N = 4$: error e_i (top) and input/output signals χ_i (bottom).
Vehicle 1 is –, Vehicle 2 is –, Vehicle 3 is –, Vehicle 4 is –.

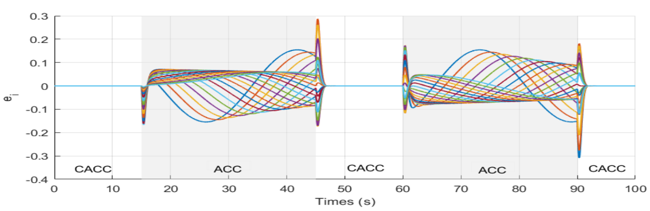

Fig. 4: Overview of platoon dynamic response error e_i for $N = 20$ where each color correspond to a vehicle.

long enough, the platoon absorbs the disturbances caused by a change of mode. In other words, the results on H_∞ specifications in Theorem 2 and interpreted in terms of switching string stability reported in Theorem 3 are also verified. For a platoon with synchronized mode change for all vehicles, we notice that the string stability is momentarily lost when switching. Each vehicle from the sets shown in Fig. 3 and Fig. 4 must not only reposition itself in relation to the vehicle directly in front, but must also reposition itself in relation to the rest of the vehicles in front. The dwell time is the minimum time required to absorb this mode change disturbance, not to be confused with disturbances caused by the speed profile of the first vehicle in the platoon. These disturbances are well reduced throughout the platoon, whatever the mode. In future work, we will consider a sequential switching strategy that does not amplify the disturbance at switching time.

VI. Conclusion

This paper provides dwell time conditions guaranteeing the strong global recurrence and H_∞ specifications for a switched system with lower bounded jumps in the state. These results are applied to analyze the string stability of homogeneous platoons subject to switches in control mode between ACC and CACC. Numerical simulations validate the approach taken here. Future work could focus on reducing conservatism and extending the approach to asynchronized heterogeneous platoons with communication delays.

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