# System Identification

Control Engineering EN, 3rd year B.Sc.
Technical University of Cluj-Napoca
Romania

Lecturer: Lucian Buşoniu

# Part V

# ARX identification

# Classification

Recall **taxonomy of models** from Part I:

By number of parameters:

1. Parametric models: have a fixed form (mathematical formula), with a known, often small number of parameters
2. Nonparametric models: cannot be described by a fixed, small number of parameters
   Often represented as graphs or tables

By amount of prior knowledge ("color"):

1. First-principles, white-box models: fully known in advance
2. Black-box models: entirely unknown
3. Gray-box models: partially known

The ARX method produces *parametric*, polynomial models.

# Why ARX?

- General-order, fully implementable method with guarantees – like correlation analysis
- Unlike correlation analysis, gives a *compact* model with a number of parameters proportional to the order of the system

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We stay in the single-output, single-input case for the entire lecture except the optional appendix. Nonlinear ARX is for the project, we won't need it for the labs.

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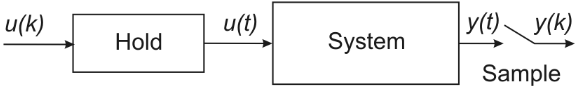
# Recall: Discrete time

We remain in the discrete-time setting:



# ARX model structure

In the ARX model structure, the output $y(k)$ at the current discrete time step is computed based on previous input and output values:

$$\begin{aligned} y(k) + a_1 y(k-1) + a_2 y(k-2) + \ldots + a_{na} y(k-na) \\ = b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb) + e(k) \end{aligned}$$

equivalent to

$$\begin{aligned} y(k) = -a_1 y(k-1) - a_2 y(k-2) - \ldots - a_{na} y(k-na) \\ b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb) + e(k) \end{aligned}$$

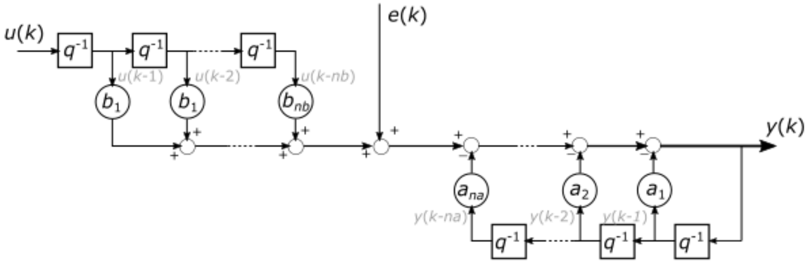
$e(k)$ is the noise at step $k$.

Model parameters: $a_1, a_2, \ldots, a_{n_a}$ and $b_1, b_2, \ldots, b_{nb}$.

Name: AutoRegressive ($y(k)$ a function of previous $y$ values) with eXogenous input (dependence on $u$)

# Graphical form

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) - \ldots - a_{na} y(k-na)$$
$$b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb) + e(k)$$




where the backward shift operator $q^{-1}$ delays any discrete-time signal $z(k)$ by one step:

$$q^{-1} z(k) = z(k-1)$$

# Polynomial ARX form

Using $q^{-1}$, we write:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + \ldots + a_{na} y(k-na)$$
$$= (1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_{na} q^{-na}) y(k) =: A(q^{-1}) y(k)$$

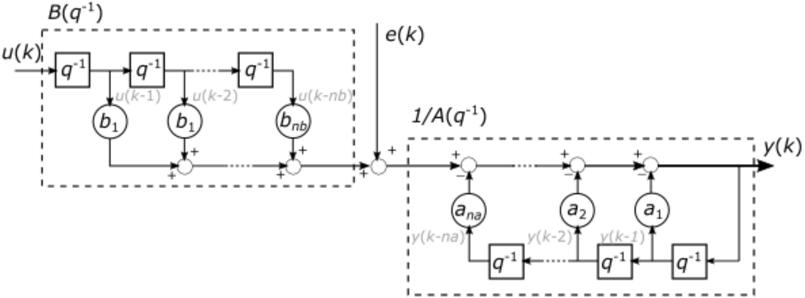
and:

$$b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb)$$
$$= (b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{nb} q^{-nb}) u(k) =: B(q^{-1}) u(k)$$

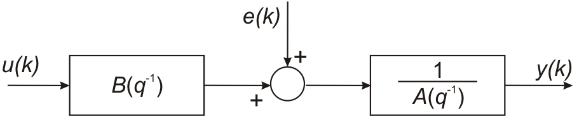
Therefore, the ARX model is written compactly:

$$\boxed{A(q^{-1}) y(k) = B(q^{-1}) u(k) + e(k)}$$

# Compact graphical form




Zooming out:




related to:

$$y(k) = \frac{1}{A(q^{-1})}[B(q^{-1})u(k) + e(k)]$$

# Remarks

1. The ARX model is quite general, it can describe arbitrary linear relationships between inputs and outputs. However, the noise enters the model in a restricted way, and later we introduce models that generalize this.

2. In the absence of noise, the model reduces to a standard discrete-time transfer function.

# Linear regression model

Returning to the explicit representation:

$$
\begin{aligned}
y(k) = & - a_1 y(k-1) - a_2 y(k-2) - \ldots - a_{na} y(k-na) \\
& b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb) + e(k) \\
= & [-y(k-1), \ldots, -y(k-na), u(k-1), \ldots, u(k-nb)] \\
& \cdot [a_1, \ldots, a_{na}, b_1, \ldots, b_{nb}]^\top + e(k) \\
=: & \varphi^\top(k)\theta + e(k)
\end{aligned}
$$

So in fact ARX obeys the standard model structure in linear regression!

# Vectors of regressors and parameters

Regressor vector: $\varphi(k) \in \mathbb{R}^{na+nb}$, previous output and input values.

Parameter vector: $\theta \in \mathbb{R}^{na+nb}$, polynomial coefficients.

$$\varphi(k) = \begin{bmatrix} -y(k-1) \\ \vdots \\ -y(k-na) \\ u(k-1) \\ \vdots \\ u(k-nb) \end{bmatrix} \qquad \theta = \begin{bmatrix} a_1 \\ \vdots \\ a_{na} \\ b_1 \\ \vdots \\ b_{nb} \end{bmatrix}$$

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# Identification problem

Consider now that we are given a dataset $u(k), y(k)$, $k = 0, \ldots, N$, and we have to find the model parameters $\theta$.

Then for any $k \geq 1$:

$$y(k) = \varphi^\top(k)\theta + \varepsilon(k)$$

where $\varepsilon(k)$ is now interpreted as an equation error (hence the changed notation).

Objective: minimize the mean squared error:

$$V(\theta) = \frac{1}{N}\sum_{k=1}^{N} \varepsilon(k)^2$$

Remark: When $k \leq na, nb$, negative-time values for $u$ and $y$ are needed to construct $\varphi$. They can be taken equal to 0 (assuming the system is in zero initial conditions).

# Linear system of equations

$$y(1) = \begin{bmatrix} -y(0) & \cdots & -y(1-na) & u(0) & \cdots & u(1-nb) \end{bmatrix} \theta$$
$$y(2) = \begin{bmatrix} -y(1) & \cdots & -y(2-na) & u(1) & \cdots & u(2-nb) \end{bmatrix} \theta$$
$$\cdots$$
$$y(N) = \begin{bmatrix} -y(N-1) & \cdots & -y(N-na) & u(N-1) & \cdots & u(N-nb) \end{bmatrix} \theta$$

Matrix form:

$$\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} -y(0) & \cdots & -y(1-na) & u(0) & \cdots & u(1-nb) \\ -y(1) & \cdots & -y(2-na) & u(1) & \cdots & u(2-nb) \\ \vdots & \vdots & \vdots & \vdots & & \\ -y(N-1) & \cdots & -y(N-na) & u(N-1) & \cdots & u(N-nb) \end{bmatrix} \cdot \theta$$

$$Y = \Phi\theta$$

with notations $Y \in \mathbb{R}^N$ and $\Phi \in \mathbb{R}^{N\times(na+nb)}$.

# ARX solution

From linear regression, to minimize $\frac{1}{2}\sum_{k=1}^{N}\varepsilon(k)^2$ the parameters are:

$$\boxed{\widehat{\theta} = (\Phi^\top \Phi)^{-1}\Phi^\top Y}$$

Since the new $V(\theta) = \frac{1}{N}\sum_{k=1}^{N}\varepsilon(k)^2$ is proportional to the criterion above, the same solution also minimizes $V(\theta)$.

# Solution for large datasets

When the number of data points $N$ is very large, the form above is impractical. In that case, a better form is the alternative one we introduced in the linear regression lecture:

$$\Phi^\top \Phi = \sum_{k=1}^{N} \varphi(k)\varphi^\top(k), \quad \Phi^\top Y = \sum_{k=1}^{N} \varphi(k)y(k)$$

$$\Rightarrow \widehat{\theta} = \left[\sum_{k=1}^{N} \varphi(k)\varphi^\top(k)\right]^{-1} \left[\sum_{k=1}^{N} \varphi(k)y(k)\right]$$

# Solution for large datasets (continued)

Remaining issue: the sum of $N$ terms can grow very large, leading to numerical problems: (matrix of very large numbers)$^{-1}\cdot$ vector of very large numbers.

Solution: Normalize element values by diving them by $N$. In equations, $N$ simplifies so it has no effect on the analytical development, but in practice it keeps the numbers reasonable.

$$\widehat{\theta} = \left[ \frac{1}{N} \sum_{k=1}^{N} \varphi(k)\varphi^\top(k) \right]^{-1} \left[ \frac{1}{N} \sum_{k=1}^{N} \varphi(k)y(k) \right]$$

What about the division by $N$? It can be implemented recursively, without ever computing large numbers – details later in the course.

# Using the model

One-step ahead prediction $\hat{y}$: The true output sequence is known, so all the delayed signals are available and we can simply plug them in the formula, together with the coefficients taken from $\theta$:

$$\hat{y}(k) = -a_1 y(k-1) - a_2 y(k-2) - \ldots - a_{na} y(k-na)$$
$$b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb)$$

Signals at negative time can be taken equal to 0.

Example: On day $k-1$, predict weather for day $k$.

Simulation $\tilde{y}$: True outputs $y(k-i)$ unknown, so we must use previously simulated outputs $\tilde{y}(k-i)$:

$$\tilde{y}(k) = -a_1 \tilde{y}(k-1) - a_2 \tilde{y}(k-2) - \ldots - a_{na} \tilde{y}(k-na)$$
$$b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb)$$

(simulated outputs at negative and zero time can also be taken 0.)

Example: Simulation of an aircraft's response to emergency pilot inputs, that may be dangerous to apply to the real system.

# Note on using models

We can run many types of of models, not just ARX, in prediction or simulation modes. This is a general concept that does not only apply to ARX.

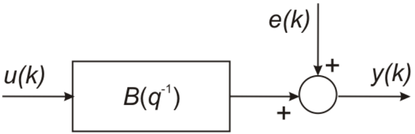
# Special case of ARX: FIR

Setting $A = 1$ ($na = 0$) in ARX, we get:

$$y(k) = B(q^{-1})u(k) + e(k) = \sum_{j=1}^{nb} b_j u(k-j) + e(k)$$
$$= \sum_{j=0}^{M-1} h(j)u(k-j) + e(k)$$

the FIR model from correlation analysis!

To see this, take $nb = M - 1$, and $b_j = h(j)$. Note $h(0)$,the impulse response at time 0, is assumed 0 – i.e. system does not respond instantaneously to changes in input.



# Fundamental difference between ARX and FIR

$$\begin{aligned} \text{ARX:} \quad & A(q^{-1})y(k) = B(q^{-1})u(k) + e(k) \\ \text{FIR:} \quad & y(k) = B(q^{-1})u(k) + e(k) \end{aligned}$$

Since ARX includes relationships between current and previous outputs, it will be sufficient to take orders *na* and *nb* equal to the order of the dynamical system.

FIR needs a sufficiently large order *nb* (or length *M*) to model the entire transient regime of the impulse response (in principle, we only recover the correct model as $M \to \infty$).

$\Rightarrow$ more parameters $\Rightarrow$ more data needed to identify them.

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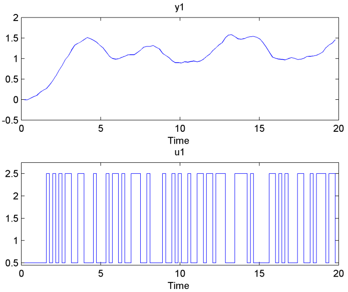
# Experimental data

Consider we are given the following, separate, identification and validation data sets.

`plot(id);` and `plot(val);`




Remarks: Identification input: a so-called *pseudo-random binary signal*. Validation input: a sequence of steps.

# Identifying an ARX model

```
model = arx(id, [na, nb, nk]);
```

Arguments:

1. Identification data.
2. Array containing the orders of $A$ and $B$ and the *delay* $nk$.

Structure different from theory: includes explicitly a minimum delay $nk$ between inputs and outputs, useful for systems with time delays.

$$\begin{aligned} y(k) + a_1 y(k-1) + a_2 y(k-2) + \ldots + a_{na} y(k-na) \\ = b_1 u(k-nk) + b_2 u(k-nk-1) + \ldots + b_{nb} u(k-nk-nb+1) + e(k) \end{aligned}$$

$$A(q^{-1}) y(k) = B(q^{-1}) u(k-nk) + e(k), \text{ where:}$$

$$A(q^{-1}) = (1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_{na} q^{-na})$$

$$B(q^{-1}) = (b_1 + b_2 q^{-1} + b_{nb} q^{-nb+1})$$

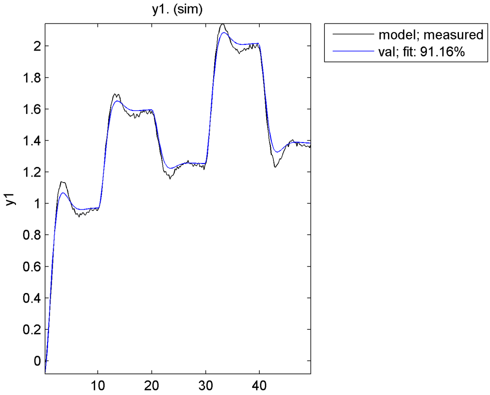
The theoretical structure is obtained by setting $nk = 1$. For $nk > 1$, we can also transform the new structure into the theoretical one by using a $B$ polynomial of order $nk + nb - 1$, with $nk - 1$ leading zeros:

$$B_{\text{theor}}(q^{-1}) = 0q^{-1} + \ldots 0q^{-nk+1} + b_1 q^{-nk} + \ldots + b_{nb} q^{-nk-nb+1}$$

# Model validation

Assuming the system is second-order, *in the ARX form*, and without time delay, we take $na = 2, nb = 2, nk = 1$. Validation:

```
compare(model, val);
```




Results are quite bad.

# Structure selection

Alternate idea: try many different structures and choose the best one.

```
Na = 1:15;
Nb = 1:15;
Nk = 1:5;
NN = struc(Na, Nb, Nk);
V = arxstruc(id, val, NN);
```

- `struc` generates all combinations of orders in `Na, Nb, Nk`.
- `arxstruc` identifies for each combination an ARX model (on the data in 1st argument), simulates it (on the data in the 2nd argument), and returns all the MSEs on the first row of `V` (see `help arxstruc` for the format of `V`).

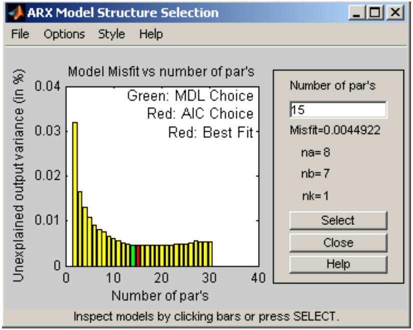
# Structure selection (continued)

To choose the structure with the smallest MSE:

```
N = selstruc(V, 0);
```

For our data, N$= [8, 7, 1]$.

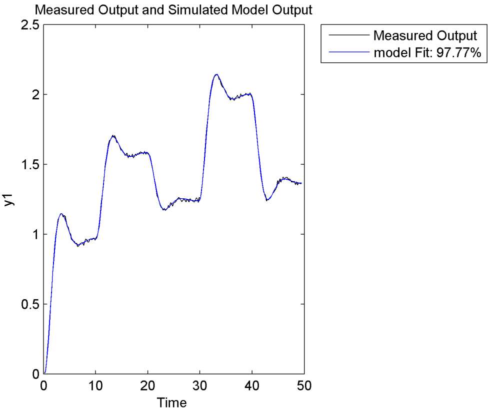
Alternatively, graphical selection: `N = selstruc(V, 'plot');`
Then click on bar corresponding to best (red) model and "Select", "Close".




(Later we learn other structure selection criteria than smallest MSE.)

# Validation of best ARX model

```
model = arx(id, N); compare(model, val);
```




A better fit is obtained. However, 8th order systems are rare in real life, so something else is likely going on... we will see later.

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# Main result

## Assumptions

1. There exists a true parameter vector $\theta_0$ so that:

$$y(k) = \varphi^\top(k)\theta_0 + v(k)$$

   with $v(k)$ a stationary stochastic process independent from $u(k)$.
2. $\mathrm{E}\left\{\varphi(k)\varphi^\top(k)\right\}$ is a nonsingular matrix.
3. $\mathrm{E}\left\{\varphi(k)v(k)\right\} = 0$.

## Theorem

ARX identification is consistent: the estimated parameters $\widehat{\theta}$ converge to the true parameters $\theta_0$, in the limit as $N \to \infty$.

# Discussion of assumptions

1. Assumption 1 is equivalent to the existence of true polynomials $A_0(q^{-1})$, $B_0(q^{-1})$ so that:

$$A_0(q^{-1})y(k) = B_0(q^{-1})u(k) + v(k)$$

To motivate Assumption 2, recall

$$\widehat{\theta} = \left[\frac{1}{N}\sum_{k=1}^{N} \varphi(k)\varphi^\top(k)\right]^{-1} \left[\frac{1}{N}\sum_{k=1}^{N} \varphi(k)y(k)\right]$$

As $N \to \infty$, $\frac{1}{N}\sum_{k=1}^{N} \varphi(k)\varphi^\top(k) \to \mathrm{E}\left\{\varphi(k)\varphi^\top(k)\right\}$.

2. $\mathrm{E}\left\{\varphi(k)\varphi^\top(k)\right\}$ is nonsingular if the data is "sufficiently informative" (e.g., $u(k)$ should not be a simple feedback from $y(k)$; see Söderström & Stoica for more discussion).
3. $\mathrm{E}\left\{\varphi(k)v(k)\right\} = 0$ e.g. if $v(k)$ is white noise. Later on, we will discuss in more detail Assumption 3 and the role of $\mathrm{E}\left\{\varphi(k)v(k)\right\} = 0$.

# Summary

- ARX model structure and representation with polynomials in $q^{-1}$
- Linear-regression form with regressors $\phi$ and parameters $\theta$
- Linear-regression, least-squares solution. Rewriting for large datasets
- Using the model for prediction and simulation
- Relationship with FIR

- Matlab example.
- Simplified accuracy guarantee.

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# Nonlinear ARX structure

Recall standard ARX:

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) - \ldots - a_{na} y(k-na)$$
$$b_1 u(k-1) + b_2 u(k-2) + \ldots + b_{nb} u(k-nb) + e(k)$$

Linear dependence on delayed outputs $y(k-1), \ldots, y(k-na)$ and inputs $u(k-1), \ldots, u(k-nb)$.

Nonlinear ARX (NARX) generalizes this to any nonlinear dependence:

$$y(k) = g\big(y(k-1), y(k-2), \ldots, y(k-na), u(k-1), u(k-2), \ldots, u(k-nb); \theta\big) + e(k)$$

Function $g$ is parameterized by $\theta \in \mathbb{R}^n$, and these parameters can be tuned to fit identification data and thereby model a particular system.

# Polynomial NARX

In our particular case, $g$ is a polynomial of degree $m$ in the delayed outputs and inputs:

$$y(k) = p(y(k-1), \dots, y(k-na), u(k-1), \dots, u(k-nb)) + e(k)$$
$$=: p(d(k)) + e(k)$$

where $d(k) = [y(k-1), \dots, y(k-na), u(k-1), \dots, u(k-nb)]^\top$ is the vector of delayed signals.

E.g., for orders $na = nb = 1$ (so $d(k) = [y(k-1), u(k-1)]^\top$) and degree $m = 1$, the model is:

$$y(k) = ay(k-1) + bu(k-1) + c + e(k)$$

which by further taking $c = 0$ recovers the linear ARX form

# Polynomial NARX (continued)

For the same $na = nb = 1$ and degree $m = 2$:

$$\begin{aligned} y(k) = & ay(k-1) + bu(k-1) + cy(k-1)^2 \\ & + du(k-1)^2 + wu(k-1)y(k-1) + z + e(k) \end{aligned}$$

- Do not confuse with polynomial form $A(q^{-1})y(k) = B(q^{-1})u(k) + e(k)$
- The parameters are now the coefficients of the polynomial, e.g. $\theta = [a, b, c, d, w, z]^\top$
- Linear regression works as usual, finding the parameters that minimize the MSE!
- Negative and zero-time $y$ and $u$ can be taken 0, assuming system in zero initial conditions

# Recall prediction versus simulation

One-step ahead prediction $\widehat{y}$: True output sequence is known, delays vector $d(k)$ is fully available:

$$d(k) = [y(k-1), \dots, y(k-na), u(k-1), \dots, u(k-nb)]^\top$$
$$\widehat{y}(k) = g(d(k); \widehat{\theta})$$

Simulation $\tilde{y}$: True outputs unknown, use the previously simulated outputs to construct an *approximation* $\tilde{d}(k)$ of $d(k)$:

$$\tilde{d}(k) = [\tilde{y}(k-1), \dots, \tilde{y}(k-na), u(k-1), \dots, u(k-nb)]^\top$$
$$\tilde{y}(k) = g(\tilde{d}(k); \widehat{\theta})$$

# Appendix: Multiple inputs and outputs

# MIMO system

So far we considered $y(k) \in \mathbb{R}$, $u(k) \in \mathbb{R}$,
*Single-Input, Single-Output (SISO)* systems

Many systems are Multiple-Input, Multiple-Output (MIMO).
E.g., aircraft. Inputs: throttle, aileron, elevator, rudder.
Outputs: airspeed, roll, pitch, yaw.

# MIMO ARX

Consider next $y(k), e(k) \in \mathbb{R}^{ny}$, $u(k) \in \mathbb{R}^{nu}$. MIMO ARX model:

$$A(q^{-1})y(k) = B(q^{-1})u(k) + e(k)$$
$$A(q^{-1}) = I + A_1 q^{-1} + \ldots + A_{na} q^{-na}$$
$$B(q^{-1}) = B_1 q^{-1} + \ldots + B_{nb} q^{-nb}$$

where $I$ is the $ny \times ny$ identity matrix, $A_1, \ldots, A_{na} \in \mathbb{R}^{ny \times ny}$, $B_1, \ldots, B_{nb} \in \mathbb{R}^{ny \times nu}$.

# Concrete example

Take $na = 1$, $nb = 2$, $ny = 2$, $nu = 3$. Then:

$$
\begin{aligned}
A(q^{-1})y(k) &= B(q^{-1})u(k) + e(k) \\
A(q^{-1}) &= I + A_1 q^{-1} \\
&= I + \begin{bmatrix} a_1^{11} & a_1^{12} \\ a_1^{21} & a_1^{22} \end{bmatrix} q^{-1} \\
B(q^{-1}) &= B_1 q^{-1} + B_2 q^{-2} \\
&= \begin{bmatrix} b_1^{11} & b_1^{12} & b_1^{13} \\ b_1^{21} & b_1^{22} & b_1^{23} \end{bmatrix} q^{-1} + \begin{bmatrix} b_2^{11} & b_2^{12} & b_2^{13} \\ b_2^{21} & b_2^{22} & b_2^{23} \end{bmatrix} q^{-2}
\end{aligned}
$$

# Concrete example (continued)

$$\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}a_1^{11} & a_1^{12}\\ a_1^{21} & a_1^{22}\end{bmatrix} q^{-1}\right)\begin{bmatrix}y_1(k)\\ y_2(k)\end{bmatrix}$$

$$= \left(\begin{bmatrix}b_1^{11} & b_1^{12} & b_1^{13}\\ b_1^{21} & b_1^{22} & b_1^{23}\end{bmatrix} q^{-1} + \begin{bmatrix}b_2^{11} & b_2^{12} & b_2^{13}\\ b_2^{21} & b_2^{22} & b_2^{23}\end{bmatrix} q^{-2}\right)\begin{bmatrix}u_1(k)\\ u_2(k)\\ u_3(k)\end{bmatrix} + \begin{bmatrix}e_1(k)\\ e_2(k)\end{bmatrix}$$

Explicit relationship:

$$\begin{aligned} y_1(k) &+ a_1^{11} y_1(k-1) + a_1^{12} y_2(k-1)\\ &= b_1^{11} u_1(k-1) + b_1^{12} u_2(k-1) + b_1^{13} u_3(k-1)\\ &+ b_2^{11} u_1(k-2) + b_2^{12} u_2(k-2) + b_2^{13} u_3(k-2) + e_1(k)\\ y_2(k) &+ a_1^{21} y_1(k-1) + a_1^{22} y_2(k-1)\\ &= b_1^{21} u_1(k-1) + b_1^{22} u_2(k-1) + b_1^{23} u_3(k-1)\\ &+ b_2^{21} u_1(k-2) + b_2^{22} u_2(k-2) + b_2^{23} u_3(k-2) + e_2(k) \end{aligned}$$

# Matlab example

Consider a continuous stirred-tank reactor:

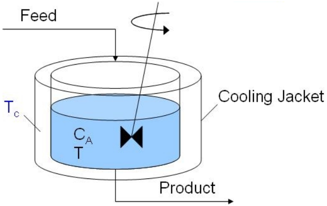



Image credit: mathworks.com

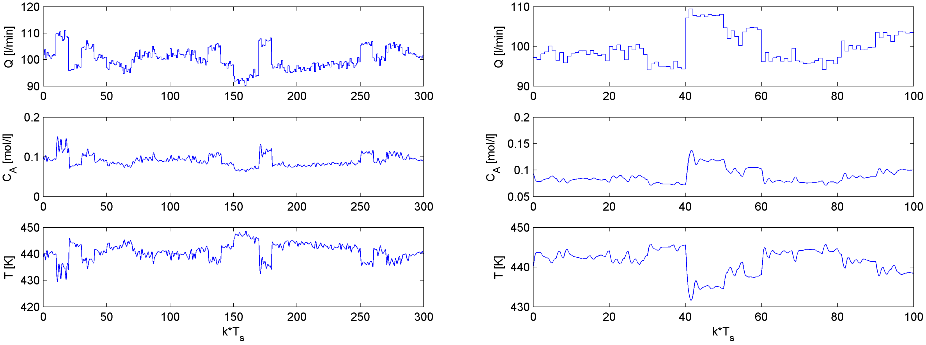
**Input:** coolant flow $Q$

**Outputs:**

- Concentration $C_A$ of substance $A$ in the mix
- Temperature $T$ of the mix

# Matlab: Experimental data

Left: identification, Right: validation



# Matlab: MIMO ARX, different from theory

$$A(q^{-1})y(k) = B(q^{-1})u(k) + e(k)$$

$$A(q^{-1}) = \begin{bmatrix} a^{11}(q^{-1}) & a^{12}(q^{-1}) & \dots & a^{1ny}(q^{-1}) \\ a^{21}(q^{-1}) & a^{22}(q^{-1}) & \dots & a^{2ny}(q^{-1}) \\ \dots & & & \\ a^{ny1}(q^{-1}) & a^{ny2}(q^{-1}) & \dots & a^{nyny}(q^{-1}) \end{bmatrix}$$

$$a^{ij}(q^{-1}) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} + a_1^{ij} q^{-1} + \ldots + a_{na_{ij}}^{ij} q^{-na_{ij}}$$

$$B = \begin{bmatrix} b^{11}(q^{-1}) & b^{12}(q^{-1}) & \dots & b^{1nu}(q^{-1}) \\ b^{21}(q^{-1}) & b^{22}(q^{-1}) & \dots & b^{2nu}(q^{-1}) \\ \dots & & & \\ b^{ny1}(q^{-1}) & b^{ny2}(q^{-1}) & \dots & b^{nynu}(q^{-1}) \end{bmatrix}$$

$$b^{ij}(q^{-1}) = b_1^{ij} q^{-nk_{ij}} + \ldots + b_{nb_{ij}}^{ij} q^{-nk_{ij} - nb_{ij} + 1}$$

# Matlab: Identifying the model

```
m = arx(id, [Na, Nb, Nk]);
```

## Arguments:

1. Identification data.
2. Matrices with orders of polynomials in *A*, *B*, and *delays nk*:

$$Na = \begin{bmatrix} na_{11} & \dots & na_{1ny} \\ \dots & & \\ na_{ny1} & \dots & na_{nyny} \end{bmatrix}$$

$$Nb = \begin{bmatrix} nb_{11} & \dots & nb_{1nu} \\ \dots & & \\ nb_{ny1} & \dots & nb_{nynu} \end{bmatrix}$$

$$Nk = \begin{bmatrix} nk_{11} & \dots & nk_{1nu} \\ \dots & & \\ nk_{ny1} & \dots & nk_{nynu} \end{bmatrix}$$

# Matlab: Results

Take $na = 2$, $nb = 2$, and $nk = 1$ everywhere in matrix elements:

```
Na = [2 2; 2 2]; Nb = [2; 2]; Nk = [1; 1];
m = arx(id, [Na Nb Nk]);
compare(m, val);
```