System Identification – Practical Assignment 3
Linear Regression for Function Approximaton

Logistics

- This practical assignment should be carried out by each student separately, if at all possible. In extremis, only if there are more students than computers, students may team up in groups of 2.
- The assignment solution consists of Matlab code. Develop this code in a single Matlab script. This code will be checked and run by the teacher during the lab class, and your attendance to the lab will only be registered if you have a working, original solution. Validated attendances for all the labs are necessary for eligibility to the exam. Moreover, at most two labs can be recovered at the end of the semester, which means accumulating three or more missing labs at any point during the semester automatically leads to final ineligibility.
- Discussing ideas amongst the students is encouraged; however, directly sharing and borrowing pieces of code is forbidden, and any violation of this rule will lead to disqualification of the solution.

Assignment description

In this assignment we will perform function approximation with linear regression and radial basis functions, see *Linear Regression* in the course material *Part 3 – Mathematical Background*.

A data set of input-output pairs is given, where the outputs are generated by an unknown function g. The function has one input variable and one output variable, and the output measurements are affected by noise. You will develop an approximator of this function, using a linear model with radial, Gaussian basis functions. The parameters of the model will be found using the identification data set. A second data set is provided for validating the developed model. The two data sets are given in a MATLAB data file, containing one structure for each set. The training data set is named `id` and the validation data set `val`. Each of these structures contains a vector X of input samples, and the corresponding output samples in vector Y.

Each student is assigned an index number. Then, the student downloads the data file that form the basis of the assignment from the course webpage:

`http://busoniu.net/teaching/sysid2018`

Requirements:

- Plot the identification data to get an idea of the function shape.
- Create a grid of radial basis functions with centers c_i on an equidistant grid defined over the same interval as the data, and having all the same radius b. The number n of basis functions (i.e. the number of points on the grid) and the radius b should be tunable.
- For given n and b, create a system of linear equations for linear regression, using the identification data. Use the matrix representation explained in the lecture. Solve this system using matrix left division, operator $\backslash$ in Matlab (or alternatively with `linsolve`). Report the MSE on the identification data.

- Validate the model on the different, validation data set: compute the approximated outputs and from those the MSE on the validation data. Show a plot of the approximated function on the validation data set, comparing to the actual outputs.
- Tune the number n of RBFs for good performance (always computed on the different, validation data set to avoid overfitting). Hint: initially, you can take the radius b to be equal to the distance between two adjacent RBFs, to obtain a smooth approximator. Then, you may optionally also tune b to improve the performance.

Your plots will look similar to those exemplified in the next figure (except your data and fit quality may be different, of course).