Reinforcement Learning in Continuous State and Action Spaces

Lucian Buşoniu

Advisers: Robert Babuška, Bart De Schutter

Delft University of Technology
Center for Systems and Control
Project: Interactive Collaborative Information Systems

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Motivation and focus

Learning can find solutions that:

- are hard to determine a priori
- improve over time

Reinforcement learning:

- uses reward signal as performance feedback
- can work without prior knowledge

Exact RL solutions only in discrete cases:

- this thesis: continuous spaces
- using approximate solutions

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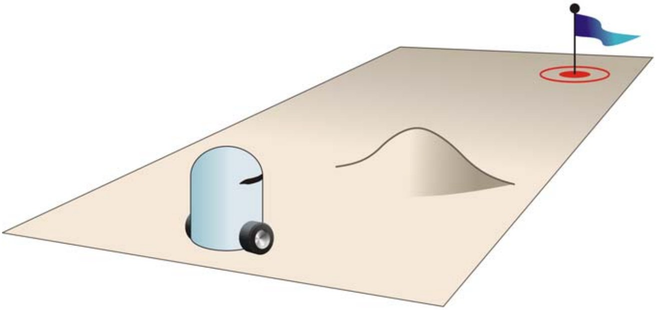
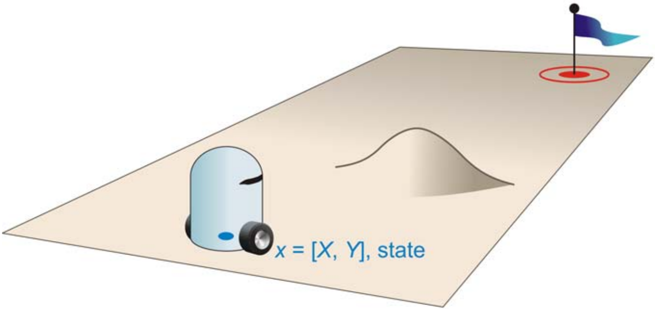
Outline

RL problem



- Optimal control problem
- Example: robot should move to goal in shortest time

Elements of RL



- Robot in given state (position X, Y)
- Robot takes action (e.g., move forward) and reaches new state
- Receives reward = quality of state transition

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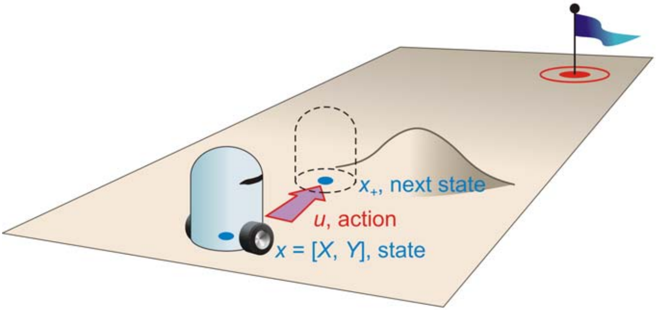

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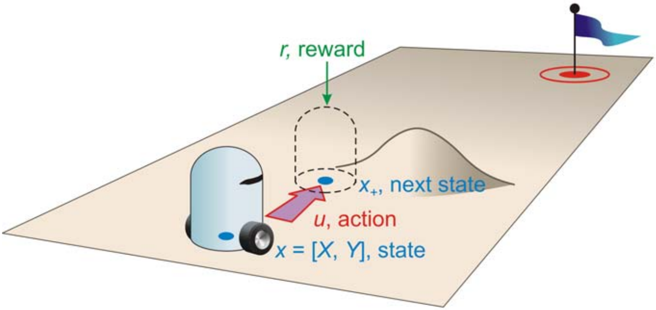

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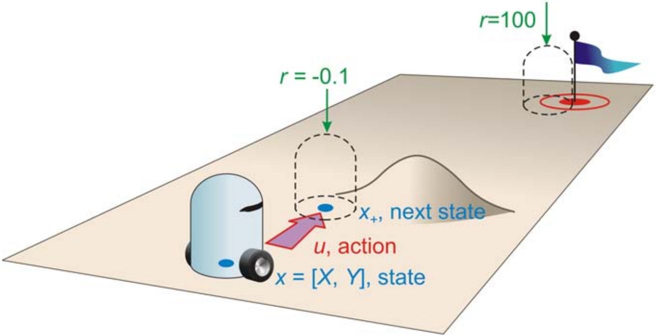

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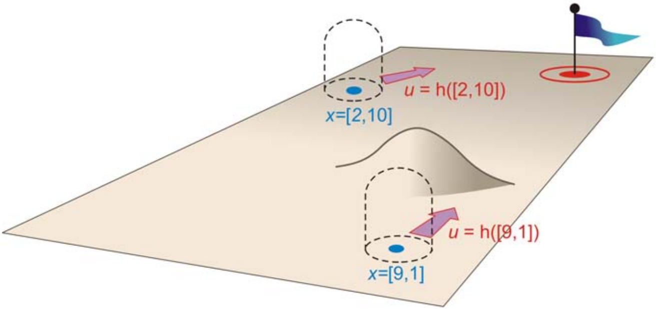

- Control policy: what action to take in every state

$$u = h(x)$$

- E.g., in state [9, 1], move forward
  in state [2, 10], move right

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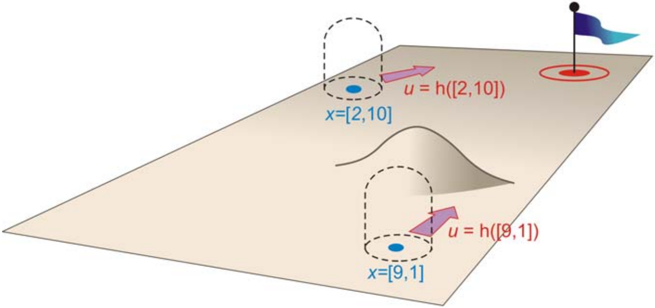

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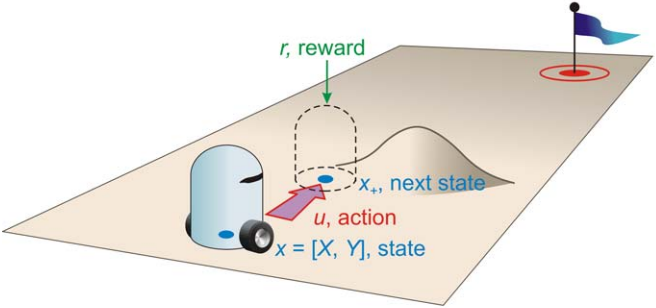
Performance criterion



- Reward = one-step performance
- Return = long-term performance, along trajectory

$$R(x, u) = r_1 + \gamma r_2 + \gamma^2 r_3 + \gamma^3 r_4 + \dots$$

- $0 < \gamma < 1$ discount factor

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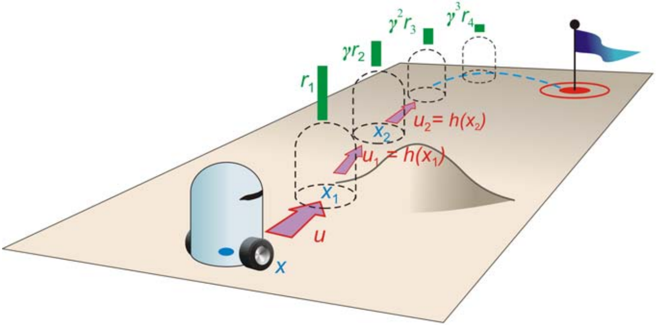

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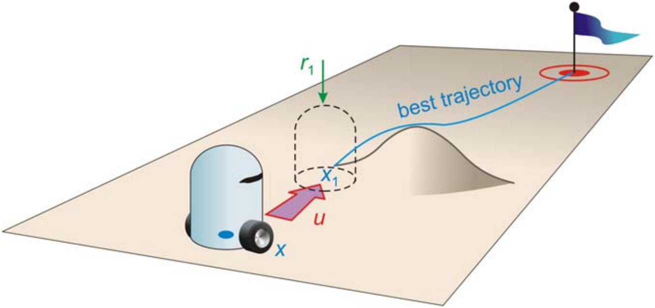

- Goal: obtain maximal return:
  $R^*(x,u) = r_1 +$ discounted rewards
  along best trajectory starting in x_1
- Optimal policy h^* can be computed from R^*

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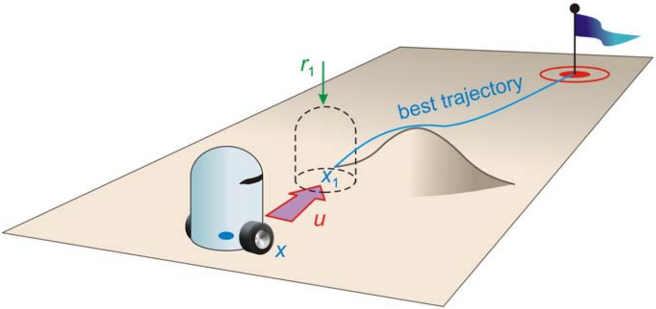

- Goal: obtain maximal return:
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Algorithms

- Many algorithms available to find optimal R^*, h^*
- Some require prior knowledge about problem, or data:
- Others work with no prior knowledge,
  and collect data by online interaction:

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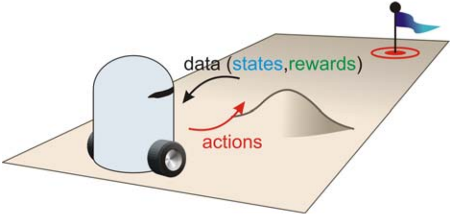
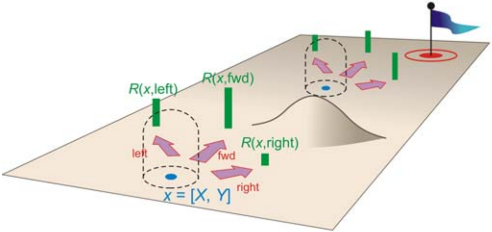


1. Reinforcement learning
2. Challenge & contribution
3. Examples
4. Conclusions

Challenge

- Classical algorithms have to store $R(x, u)$ for every combination of state x and action u



- Only possible for when number of combinations is small
- However, x and u often continuous
  $\Rightarrow$ infinitely many combinations!
- E.g., for robot, $x = [X, Y]$ continuous

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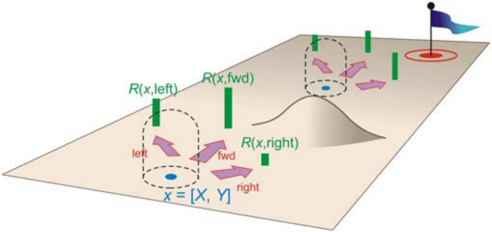

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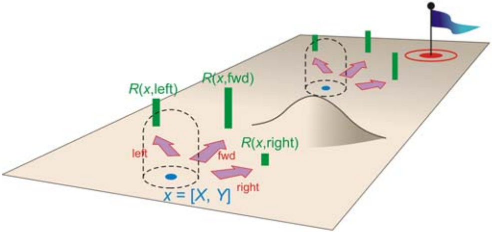

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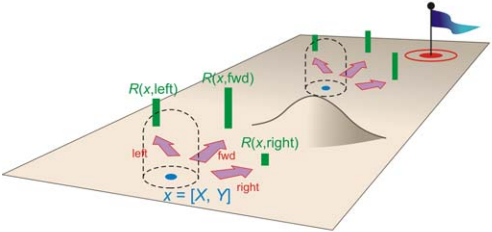

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Contribution

- Algorithms for reinforcement learning
  in problems with continuous states and actions
- Using approximate representations of returns

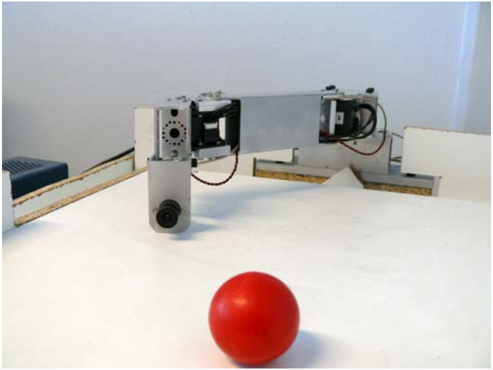
Example: RL for inverted pendulum

- Goal: point up and stay there
- Difficulty: insufficient power, need to swing back & forth
- Reward: the closer to vertical, the larger

Replay learned policy

Example: RL for robot goalkeeper

- Catch ball using only video camera image



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- Reinforcement learning: very general framework
- Can learn from interaction, without prior knowledge

However:

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Application areas

1. Control engineering: optimal & learning control (e.g., robot control)
2. Computer science: intelligent agents
3. Economics
4. etc.

Thank you

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Questions?