Robust Observer-Based Tracking Control Design for Power-Assisted Wheelchairs

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Abstract: Power-assisted wheelchairs (PAW) are an efficient mean of transport for disabled persons. As a human-machine system, several unknown parameters are present such as the mass of users or the ground adhesion. Moreover, the human torque signals produced are required for elaborating a robust assistive strategy and the torque sensors increase significantly the cost of the system. To solve these issues, we propose a robust observer-based controller using a polytopic representation. The closed-loop control design is composed of two steps: a state feedback controller with full state available and in a second step, an unknown input observer to estimate human torques and feed them into the obtained controller. To obtain the predefined performance, the observer gains are computed by solving a LMI problem. The goal is to guarantee the $\mathcal{H}_\infty$ estimation performance while achieving reference tracking. Finally, simulation results validate the control design. The methodology follows patent WO2015173094 issued in 2015 (Mohammad et al. 2015).

***Keywords**:* Disabled persons, Power-assisted wheelchair, Lyapunov function, Linear matrix inequality (LMI), Unknown input observer (UIO).

1. INTRODUCTION

In modern societies, rapid population aging increases considerately the number of disabled people. To enhance their mobility, power-assisted wheelchairs (PAWs) provide an efficient solution. One important advantage of PAWs is that users supply a propulsion according to their physical/metabolic constraints and the electrical motor provides a corresponding assistance to perform the driving task. This hybrid propulsion, for example the motorisation kit Duo designed by AutoNomad Mobility, start-up issued from the Lab (Mohammad et al. 2015, https://www.autonomad-mobility.com/), allows users to have a desired physical exercise.

Recently, different studies have been done for improving the manoeuvrability, safety and efficiency of PAWs. The investigations (Seki et al. 2009, Seki et al. 2011) have elaborated advanced assistive strategies to deal with the impact of different road conditions. In (Feng et al. 2018 B), a control based on optimal control theory has been designed. The objective is to enable users to have a desired fatigue variation for a predefined driving task. The experimental results have shown the efficiency of the applied model-free learning approach. However, the control strategy provided by any learning method does not guarantee formally neither the performances nor the stability of the closed loop system.

The present study relies on the patent WO2015173094 (Mohammad et al. 2015). In the present paper, a robust observer-based tracking control is proposed for the uncertain human-wheelchair system. The mass of users and the viscous friction coefficient are supposed unknown and bounded in a fixed-interval. It represents different real-time situations such that different persons using the same PAW or a varying ground profile etc. The goal is to guarantee performances for the whole set of conditions via robust control design. Moreover, users push a PAW depending on their will and their pushing techniques may not be robustly stable for the uncertain human-wheelchair system (Sehoon et al. 2014). Unstable situations are, of course, to be completely avoided in order to prevent user injuries and/or wheelchair damages. Therefore, a robust assistive strategy is required. Knowing that human input cannot be enforced, to avoid the instable situations created by users' pushing, the proposed controller needs first to compensate the influence of human torques. In particular, to remove torque sensors, the estimated human inputs computed by an unknown input observer (Guerra et al. 2015) are used directly for the human torque compensation. In addition, the controller should also guarantee a whole closed-loop stability and achieve the reference tracking objective.

Using a polytopic Takagi-Sugeno representation, the control design is formulated as a two-steps LMI optimization problems. Compared to computing the control gains and the observer gains simultaneously, the investigation (Bennani et al. 2017) shows that the two-steps LMI observer-based control design may reduce the conservativeness. In this approach, the first step is to design a state feedback robust PI (Proportional-Integral) tracking control by considering that the human torques are measured. The control gains calculated at this first step are fixed for the second step. Assuming a null n_p derivative of human torques, the observer gains are obtained by solving a LMI constraint problem. The goal is to guarantee the closed-loop stability and an $\mathcal{H}_\infty$ attenuation performance.

Since the human torques are compensated fully by the motors, the way for users to manipulate the wheelchair is to change reference signals (Feng et al. 2018 A). These references depend on the users' intention derived from the estimated torque signals. Consequently, an accurate torque estimation is not only important for tracking performance but also crucial for the manoeuvrability of the wheelchair. The observer design is part of the second step to have more flexibility to improve the performances. For the reference generation, more details can be found in (Feng et al. 2018 A).

This paper is organized as follows. In Section 2, we introduce the mathematical model of the wheelchair. Section 3 gives the control objective, the robust PI controller design and the observer-based tracking control design Section 4 provides simulation results. Section 5 gives our conclusions.

2. WHEELCHAIR MODELING

The studied wheelchair is modelled as a two-wheeled transporter, see Fig. 1. The system nomenclature used in this paper is given in Table. 1. The two-wheeled PAW is described by the dynamics (Shibata et Murakami 2008, Tsai et Hsueh 2012):

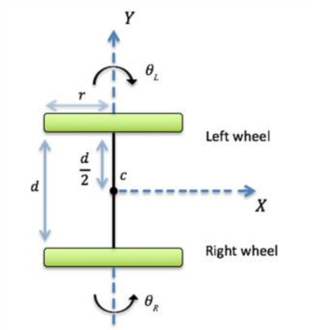

Figure 1: Simplified top view of the wheelchair

TABLE I
SYSTEM NOMENCLATURE

Symbol	Description	Value
r	Wheel radius [m]	0.33
b	Distance between two wheels [m]	0.6
c	centre of gravity of the wheelchair with the human	/
m	mass of wheelchair including the human [kg]	10
K	Viscous friction coefficient [N.m.s]	5
I_C	Inertia of the wheelchair with respect to the vertical axis through c	40
I_0	Inertia of each driving wheel around the wheel axis [kg.m^2]	0.25
T_e	Sampling time [s]	0.5

$$\alpha\dot{\theta}_R + \beta\dot{\theta}_L = T_{mr} + T_{hr} - K\theta_R \quad (1)$$
$$\alpha\dot{\theta}_L + \beta\dot{\theta}_R = T_{ml} + T_{hl} - K\theta_L$$

where the inertial parameters α and β are:

$$\alpha = \frac{mr^2}{4} + \frac{I_C r^2}{b^2} + I_0 \quad (2)$$
$$\beta = \frac{mr^2}{4} - \frac{I_C r^2}{b^2}$$

The left angular velocity and the right angular velocity are respectively θ_L and θ_R. The total torques consists of the human torques T_{hr}, T_{hl} and the assistive torques T_{mr}, T_{ml} given by the electrical motors. Using Euler's approximation $\dot{x}(t) = (x^+ - x)/T_e$, the nominal system (1) can be rewritten in the following discrete-time descriptor form:

$$E_d x^+ = A_d x + B_d u_h + B_d u_m \quad (3)$$
$$y = Cx$$

with the state vector $x^T = [\theta_R, \theta_L]$, the human torques $u_h^T = [T_{hr}, T_{hl}]$, the motor torques $u_m^T = [T_{mr}, T_{ml}]$ and the outputs $y^T = [\theta_R, \theta_L]$. The corresponding matrices are:

$$E_d = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}, A_d = \begin{bmatrix} \alpha - T_e K & \beta \\ \beta & \alpha - T_e K \end{bmatrix},$$
$$B_d = T_e I_{2\times2}, C_d = I_{2\times2}.$$

Considering that both the mass and the viscous friction coefficient are unknown and possibly time varying, uncertainties are introduced in the nominal system (3) to get the discrete-time uncertain system as follows:

$$E_d(m)x^+ = A_d(m,K)x + B_d u_h + B_d u_m \quad (4)$$
$$y = C_d x$$

As usual, the uncertainties are supposed norm bounded and sector nonlinearity applies with $\underline{m} < m < \overline{m}$ and $\underline{K} < K < \overline{K}$. Then, the uncertain system (4) can be represented by a fuzzy T-S model, i.e. a convex sum of linear models whose weights are unknown. We rewrite the system as the following polytopic representation:

$$\sum_{j=1}^{2} \varphi_j(m)\, E_{d_j} x^+ = \quad (5)$$
$$\sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K)\, A_{d_{ij}} x + B_d u_h + B_d u_m$$
$$y = C_d x$$

where φ_j are unknown convex sum weights fulfilling the conditions: $\varphi_j \in [0,1]$ $\sum_{j=1}^{2} \varphi_j(m) = 1$ and $\sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K) = 1$. The known matrices E_{d_j} and A_{d_j} are the vertices of the polytope such that:

$$E_{d_1} = E_d(\overline{m}), E_{d_2} = E_d(\underline{m}),$$
$$A_{d_{11}} = A_d(\overline{m}, \overline{K}), A_{d_{12}} = A_d(\underline{m}, \overline{K}),$$
$$A_{d_{21}} = A_d(\overline{m}, \underline{K}), A_{d_{22}} = A_d(\underline{m}, \underline{K}).$$

Using this polytopic representation and LMI techniques, we aim to provide a robust analysis for the proposed observer-based control.

Note that keeping a descriptor form, the uncertainties do not affect the input matrix B_d thus reducing the number of LMI constraints (5) (Estrada-Manzo et al. 2015). Moreover, the inversion of the non-singular matrix $E_d(m)$ is avoided. These properties are interesting to get LMI solutions using uncertainties described in (5).

3. CONTROL DESIGN

The main issue of the problem is not to measure some of the inputs of the model, i.e. the torques produced by the human. Thus, in order to estimate them to ensure a good control of the PAW, an unknown input observer is designed. Thus, from these estimations, a control law can be derived. Nevertheless, the problem we are faced to is not convex if the Lyapunov function the observer gains and the controller gains are searched all-in-one. Therefore, we decompose the problem into two steps with a guarantee of performances of the whole closed-loop.

A. Control objective

Step 1 consists in designing a robust state feedback PI-like controller assuming the states and the inputs are perfectly known. Step 2 consists in designing the observer to estimate the human torques and guaranteeing the closed-loop performances, i.e. the observer design uses a LMI constraints problem such that the uncertain system (4) with the proposed observer-based tracking controller satisfies the following requirements:

- When the reference signal $x_{ref} = 0$, the state of the uncertain system (4) and the estimation error e_{obs} converge asymptotically to the origin.
- When the reference signal $x_{ref} \neq 0$, under null initial conditions (state and estimation error), the $\mathcal{L}_2$-norm of the estimation error is bounded as follows:

$$\sum_{k=0}^{\infty} e_{obs}^T e_{obs} < \gamma \sum_{k=0}^{\infty} x_{ref}^T x_{ref}$$

B. Step 1: Robust feedback PI-like controller

In this section, states and inputs are perfectly known and we propose to design a robust PI-like controller for the uncertain system (**4**) via a LMI constraints problem. Let us recall first the classical Finsler's lemma.

Lemma 1. (De Oliveira et Skelton 2001). Let $X\epsilon\mathbb{R}^n$, $Q = Q^T\epsilon\mathbb{R}^{n\times n}$, and $W\epsilon\mathbb{R}^{m\times n}$ such that $\text{rank}(W) < n$; the following expressions are equivalent:

a) $X^Y QX < 0, \forall X\epsilon\{ X\epsilon\mathbb{R}^n : X \neq 0, WX = 0 \}$
b) $\exists M\epsilon\mathbb{R}^{n\times m} : MW + W^T M^T + Q < 0$

The control law corresponds to:

$$\begin{cases} u_m = LM^{-1}\begin{bmatrix} x \\ x_i \end{bmatrix} - u_h \\ x_i^+ = x_i - C_d x \end{cases} \tag{6}$$

where x_i corresponds to the integrator state. With the controller (6) and the uncertain system (4), the closed loop dynamic can be written as the following equality constraint:

$$\sum_{j=1}^{2} \varphi_j(m)\, \bar{E}_{c_j}\bar{x}_c = \left[\sum_{j=1}^{2}\sum_{j=1}^{2} \varphi_j(m)\,\vartheta_i(K)\,\bar{A}_{c_{ij}} + \bar{B}_c LM^{-1}\right]\bar{x}_c^- \tag{7}$$

With: $\bar{x}_c = [x \quad x_i]^T$ and the matrices:

$$\bar{E}_{c_j} = \begin{bmatrix} E_{d\,j} & 0_{2\times 2} \\ 0_{2\times 2} & I_{2\times 2} \end{bmatrix}, \bar{A}_{c_{ij}} = \begin{bmatrix} A_{d_{ij}} & 0_{2\times 2} \\ -C_d & I_{2\times 2} \end{bmatrix}, \bar{B}_c = \begin{bmatrix} B_d \\ 0_{2\times 2} \end{bmatrix}.$$

Moreover,

$$\bar{E}_c(m) = \sum_{j=1}^{2} \varphi_j(m)\, \bar{E}_{c_j} \qquad \bar{A}_c(m,K) = \sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\,\vartheta_i(K)\,\bar{A}_{c_{ij}} \tag{8}$$

Consider the following Lyapunov function candidate:

$$V(\bar{x}_c) = \bar{x}_c^T \sum_{j=1}^{2}\sum_{j=1}^{2} \varphi_j(m)\,\vartheta_i(K)\,P_{c_{ij}}\bar{x}_c \tag{9}$$

The symmetric matrices $P_{c_j} \in \mathbb{R}^{4\times 4}$, $j \in \{1,2\}$, $i \in \{1,2\}$, $h \in \{1,2\}$ are positive-definite, $P_{c_j} = P_{c\ j}^T > 0$ as well as the matrix:

$$\begin{aligned} P_c = P_c^T &= \sum_{j=1}^{2}\sum_{j=1}^{2} \varphi_j(m)\,\vartheta_i(K)\,P_{c_{ij}} \\ P_c^- = P_c^{-T} &= \sum_{j=1}^{2}\sum_{h=1}^{2} \varphi_j(m)\,\vartheta_h(K^-)\,P_{c_{hj}} \end{aligned} \tag{10}$$

Notice that only the friction is time varying, the mass even if unknown is, of course, constant for a given trial.

Theorem 1. The uncertain system (4) with the controller (6) is asymptotically stable if there exist symmetric positive-definite matrices $X_j \in \mathbb{R}^{4\times 4}$, $j \in \{1,2\}$, $i \in \{1,2\}$, $h \in \{1,2\}$, a matrix $L \in \mathbb{R}^{2\times 4}$ and a regular matrix $M \in \mathbb{R}^{4\times 4}$ such that:

$$\begin{bmatrix} -X_{hj} & (*) \\ \bar{A}_{c_{ij}}M + \bar{B}_c L & X_{ij} - \bar{E}_{c_j}M - (*) \end{bmatrix} < 0 \tag{11}$$

Proof: the variation of the Lyapunov function (9) can be written as the following inequality constraint:

$$\Delta V(\bar{x}_c) = \begin{bmatrix} \bar{x}_c^- \\ \bar{x}_c \end{bmatrix}^T \begin{bmatrix} -P_c^- & 0_{4\times 4} \\ 0_{4\times 4} & P_c \end{bmatrix} \begin{bmatrix} \bar{x}_c^- \\ \bar{x}_c \end{bmatrix} < 0 \tag{12}$$

From Lemma 1, the inequality (12) under the constraint (7) is equivalent to the inequality:

$$Q[\bar{A}_c(m,K) + \bar{B}_c LM^{-1} \quad -\bar{E}_c(m)] + (*) + \begin{bmatrix} -P_c^- & 0_{4\times 4} \\ 0_{4\times 4} & P_c \end{bmatrix} < 0 \tag{13}$$

As usual an asterisk $(*)$ represents a matrix transpose. When it is $W + (*)$ it stands for $W + W^T$, inside a matrix it represents the transpose of the entry in the symmetric position. By choosing $Q = [0_{4\times 4} \quad M^{-1}]^T$ and using the property of congruence with $diag(M^T, M^T)$, the inequality (13) is equivalent to:

$$\begin{bmatrix} -M^T P_c^- M & (*) \\ \bar{A}_c(m,K)M + \bar{B}_c L & M^T P_c M - \bar{E}_c(m)M - (*) \end{bmatrix} < 0 \tag{14}$$

Since (8), (10) and $\sum_{j=1}^{2}\sum_{j=1}^{2}\sum_{h=1}^{2} \varphi_j(m)\,\vartheta_i(K)\vartheta_h(K^-) = 1$, the inequality (14) holds if:

$$\begin{bmatrix} -M^T P_{c_{hj}} M & (*) \\ \bar{A}_{c_{ij}}M + \bar{B}_c L & M^T P_{c_{ij}} M - \bar{E}_{c_j}M - (*) \end{bmatrix} \tag{15}$$

Let $X_{hj} = M^T P_{c_{hj}} M$ and $X_{ij} = M^T P_{c_{ij}} M$, we obtain directly the linear matrix inequality (11).

C. Step 2: Observer-based tracking control design

In the previous section, we designed the controller (6) assuming the human input u_h is measured. To get rid of this assumption, we use unknown input observer to estimate the human torque. The objectives of this step are twofold; designing the observer and guaranteeing the whole closed-loop stability and performances.

A reasonable assumption for torques T_{hr} and T_{hl} estimation is to suppose that they can be approximated by a n_pth degree polynomial function in time, i.e.: $d^{n_p}T/dt^{n_p} = 0$, thus in discrete, the input torques are supposed to satisfy:

$$(1 - z^{-1})^{n_p} T_{hr}(k) = 0 \tag{16}$$

Further, (16) can be expressed as:

$$T_{hr}(k) = -\sum_{i=1}^{n_p} \binom{n_p}{i} (-1)^{n_p} T_{hr}(k-i) \tag{17}$$

where $\binom{n_p}{i}$ corresponds to the binomial coefficient. Consider the unknown input vector $T_{hr}^{n_p}(k) = \left[T_{hr}(k), T_{hr}(k-1), \dots, T_{hr}(k-n_p+1)\right]^T \epsilon \mathbb{R}^{n_p}$. The dynamic (17) of the vector $T_{hr}^{n_p}$ can be written as:

$$T_{hr}^{n_p}(k+1) = \Gamma_{n_P} T_{hr}^{n_p}(k) \tag{18}$$

where:

$$\Gamma_{n_P} = \begin{bmatrix} -(-1)^1\binom{n_p}{1} & -(-1)^2\binom{n_p}{2} & \dots & -(-1)^{n_p}\binom{n_p}{n_p} \\ & I_{n_p-1} & & 0_{(n_p-1)\times 1} \end{bmatrix}$$

The same reasoning is applied for the left wheel, the dynamic of the vector $T_{hl}^{n_p}(k) = \left[T_{hl}(k), T_{hl}(k-1), \dots, T_{hl}(k-n_p+1)\right]^T \epsilon \mathbb{R}^{n_p}$ is:

$$T_{hl}^{n_p}(k+1) = \Gamma_{n_P} T_{hl}^{n_p}(k) \tag{19}$$

Defining an extended state vector as $\bar{x}_{obs} = \left[x,\ T_{hr}^{n_p\,T}, T_{hl}^{n_p\,T}\right]^T \epsilon \mathbb{R}^{2n_p+2}$, the uncertain system (4) can be rewritten as:

$$\begin{gathered} \sum_{j=1}^{2} \varphi_j(m)\, \bar{E}_{obs\,j} \bar{x}_{obs}^{+} \\ = \sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K)\, \bar{A}_{obs_{ij}} \bar{x}_{obs} + \bar{B}_{obs} u_m \\ y = \bar{C}_{obs} \bar{x}_{obs} \end{gathered} \tag{20}$$

where:

$$\bar{E}_{obs\,j} = \begin{bmatrix} E_{d\,j} & 0_{2\times 2n_p} \\ 0_{2n_p\times 2} & I_{2n_p} \end{bmatrix}, \bar{C}_{obs} = \begin{bmatrix} C_d & 0_{2\times 2n_p} \end{bmatrix},$$

$$\bar{A}_{obs_{ij}} = \begin{bmatrix} A_{d_{ij}} & B_1 & B_2 \\ 0_{n_p\times 2} & \Gamma_{n_P} & 0_{n_p\times n_p} \\ 0_{n_p\times 2} & 0_{n_p\times n_p} & \Gamma_{n_P} \end{bmatrix}, \bar{B}_{obs} = \begin{bmatrix} B_d \\ 0_{2n_p\times 2} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \begin{bmatrix} T_e \\ 0 \end{bmatrix} & 0_{2\times(n_p-1)} \end{bmatrix}, B_2 = \begin{bmatrix} \begin{bmatrix} 0 \\ T_e \end{bmatrix} & 0_{2\times(n_p-1)} \end{bmatrix}.$$

Based on the nominal system (3), the observer is considered as follows:

$$\bar{E}_{obs}^{*} \hat{\bar{x}}_{obs}^{+} = \bar{A}_{obs}^{*} \hat{\bar{x}}_{obs} + \bar{B}_{obs} u_m + G^{-1} W (y - \hat{y}) \tag{21}$$

$$\hat{y} = \bar{C}_{obs} \hat{\bar{x}}_{obs}$$

where the nominal system matrices are:

$$\bar{E}_{obs}^{*} = \begin{bmatrix} E_d & 0_{2\times 2n_p} \\ 0_{2n_p\times 2} & I_{2n_p} \end{bmatrix},$$

$$\bar{A}_{obs}^{*} = \begin{bmatrix} A_d & B_1 & B_2 \\ 0_{n_p\times 2} & \Gamma_{n_P} & 0_{n_p\times n_p} \\ 0_{n_p\times 2} & 0_{n_p\times n_p} & \Gamma_{n_P} \end{bmatrix}$$

The estimation error is $e_{obs} = \bar{x}_{obs} - \hat{\bar{x}}_{obs}$ and its dynamic is given by:

$$\begin{gathered} \bar{E}_{obs}^{*} e_{obs}^{+} + \begin{bmatrix} \sum_{j=1}^{2} \varphi_j(m)\, E_{d\,j} - E_d \\ 0_{2n_p\times 2} \end{bmatrix} x^{+} \\ = (\bar{A}_{obs}^{*} - G^{-1} W \bar{C}_{obs}) e_{obs} + \\ \begin{bmatrix} \sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K)\, A_{d_{ij}} - A_d \\ 0_{2n_p\times 2} \end{bmatrix} x \end{gathered} \tag{22}$$

The observer-based control law is computed as follows:

$$\begin{cases} u_m = L M^{-1} \begin{bmatrix} x \\ x_i \end{bmatrix} - \hat{u}_h \\ x_i^{+} = x_i - C_d x + x_{ref} \end{cases} \tag{23}$$

where x_{ref} is the reference velocity and $\hat{u}_h$ is the estimated human torques. The uncertain system (4) with the observer-based controller (21)-(23) gives the following closed loop dynamic:

$$\begin{bmatrix} \sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K) \tilde{A}_{ij} \\ \sum_{j=1}^{2} \varphi_j(m)\, \tilde{E}_j \\ -\tilde{B} \end{bmatrix}^T \begin{bmatrix} \tilde{x}^{-} \\ \tilde{x} \\ x_{ref} \end{bmatrix} = 0 \tag{24}$$

where $\tilde{x} = \left[\bar{x}_c^T \quad e_{obs}{}^T\right]^T$ and the matrices are:

$$\tilde{E}_j = \begin{bmatrix} \bar{E}_{c\,j} & 0_{4\times(2n_p+2)} \\ \begin{bmatrix} E_{d\,j} - E_d & 0_{2\times 2} \\ \multicolumn{2}{c}{0_{2n_p\times 4}} \end{bmatrix} & \bar{E}_{obs}^{*} \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0_{2\times 2} \\ I_{2\times 2} \\ 0_{(2n_p+2)\times 2} \end{bmatrix},$$

$$\tilde{A}_{ij} = \begin{bmatrix} \bar{A}_{c_{ij}} + \bar{B}_c L M^{-1} & \begin{bmatrix} 0_{4\times 2} & \begin{matrix} B_1 & B_2 \\ 0_{2\times 2} & 0_{2\times 2} \end{matrix} \end{bmatrix} \\ \begin{bmatrix} A_{d_{ij}} - A_d & 0_{2\times 2} \\ \multicolumn{2}{c}{0_{2n_p\times 4}} \end{bmatrix} & \bar{A}_{obs}^{*} - G^{-1} W \bar{C}_{obs} \end{bmatrix}.$$

Consider the following Lyapunov function candidate:

$$\begin{gathered} V(\tilde{x}) = \tilde{x}^T \sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K)\, \tilde{P}_{ij} \tilde{x} \\ V(\tilde{x}^{-}) = \tilde{x}^{-T} \sum_{j=1}^{2}\sum_{h=1}^{2} \varphi_j(m)\, \vartheta_h(K^{-})\, \tilde{P}_{hj} \tilde{x}^{-} \end{gathered} \tag{25}$$

The symmetric matrix $\tilde{P} \in \mathbb{R}^{(6+2n_p)\times(6+2n_p)}$ is positive-definite, so $\tilde{P} = \tilde{P}^T > 0$, $\tilde{P} = \sum_{j=1}^{2}\sum_{i=1}^{2} \varphi_j(m)\, \vartheta_i(K)\, \tilde{P}_{ij}$ and $\tilde{P}^{-} = \sum_{j=1}^{2}\sum_{h=1}^{2} \varphi_j(m)\, \vartheta_i(K^{-})\, \tilde{P}_{hj}$.

Theorem 2. Given matrices L and M, if there exist positive definite matrices $\tilde{P} \in \mathbb{R}^{(2n_p+6)\times(2n_p+6)}$, $j \in \{1,2\}$, $i \in \{1,2\}$, $h \in \{1,2\}$ and matrices $W \in \mathbb{R}^{(2n_p+2)\times 2}$, $G_1 \in \mathbb{R}^{4\times 4}$,

$G_2 \in \mathbb{R}^{6\times4}$, a regular matrix $G \in \mathbb{R}^{(2n_p+2)\times(2n_p+2)}$ and a positive scalar γ such that:

$$\tilde{\Pi} = \tilde{\Pi}_1 + \tilde{\Pi}_2 + \tilde{\Pi}_2^T < 0 \tag{26}$$

where

$$\tilde{\Pi}_1 = \begin{bmatrix} -\tilde{P}_{hj} + \tilde{C}^T\tilde{C} & 0_{(2n_p+6)\times(2n_p+6)} & 0_{(2n_p+6)\times 2} \\ 0_{(2n_p+6)\times(2n_p+6)} & \tilde{P}_{ij} & 0_{10\times2} \\ 0_{2\times(2n_p+6)} & 0_{2\times(2n_p+6)} & -\gamma I_{2\times2} \end{bmatrix},$$

$$\tilde{\Pi}_2 = \begin{bmatrix} \epsilon\tilde{G}\tilde{A}_{ij} & -\epsilon\tilde{G}\tilde{E}_j & -\epsilon\tilde{G}\tilde{B} \\ \tilde{G}\tilde{A}_{ij} & -\tilde{G}\tilde{E}_j & -\tilde{G}\tilde{B} \\ 0_{2\times(2n_p+6)} & 0_{2\times(2n_p+6)} & 0_{2\times2} \end{bmatrix},$$

$$\tilde{G} = \begin{bmatrix} G_1 & 0_{4\times(2n_p+2)} \\ G_2 & G \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0_{(2n_p+2)\times4} & I_{(2n_p+2)\times(2n_p+2)} \end{bmatrix}.$$

and

$$\tilde{G}\tilde{A}_{ij} = \begin{bmatrix} G_1 & 0_{4\times(2n_p+2)} \\ G_2 & I_{(2n_p+2)\times(2n_p+2)} \end{bmatrix} \begin{bmatrix} \bar{A}_{c_{ij}} + \bar{B}_c LM^{-1} & \begin{bmatrix} 0_{4\times2} & B_1 & B_2 \\ & 0_{2\times2} & 0_{2\times2} \end{bmatrix} \\ G\begin{bmatrix} A_{d_{ij}} - A_d & 0_{2\times2} \\ 0_{(2n_p+2)\times2} & \end{bmatrix} & G\bar{A}_{obs}^* - W\bar{C}_{obs} \end{bmatrix}$$

Then the observer-based tracking controller solves the control objective stated in Section 3.A.

Proof: The inequality (26) can be rewritten as follows:

$$\tilde{\Pi}_1 + \tilde{G}^*\begin{bmatrix} \tilde{A}_{ij} & -\tilde{E}_j & -\tilde{B} \end{bmatrix} + (*) < 0 \tag{27}$$

where $\tilde{G}^* = \begin{bmatrix} \epsilon\tilde{G}^T & \tilde{G}^T & 0_{2\times(2n_p+6)}^T \end{bmatrix}^T$. From (27) and $\sum_{j=1}^2\sum_{j=1}^2\sum_{h=1}^2 \varphi_j(m)\,\vartheta_i(K)\vartheta_h(K^-) = 1$, we obtain:

$$\sum_{j=1}^2\sum_{j=1}^2\sum_{h=1}^2 \varphi_j(m)\,\vartheta_i(K)\vartheta_h(K^-)\,\tilde{\Pi}_1 + \tilde{G}^*\begin{bmatrix} \sum_{j=1}^2\sum_{i=1}^2 \varphi_j(m)\,\vartheta_i(K)\tilde{A}_{ij} \\ \sum_{j=1}^2 \varphi_j(m)\,\tilde{E}_j \\ -\tilde{B} \end{bmatrix}^T + (*) < 0 \tag{28}$$

Using Lemma 1 and the constraint (21), we have:

$$\begin{bmatrix} -\tilde{P}^- + \tilde{C}^T\tilde{C} & 0_{(2n_p+6)\times(2n_p+6)} & 0_{(2n_p+6)\times2} \\ 0_{(2n_p+6)\times(2n_p+6)} & \tilde{P} & 0_{10\times2} \\ 0_{2\times(2n_p+6)} & 0_{2\times(2n_p+6)} & -\gamma I_{2\times2} \end{bmatrix} < 0 \tag{29}$$

Pre- and post-multiplying (29) with the vector $\begin{bmatrix} \tilde{x}^{-T} & \tilde{x}^T & x_{ref}^T \end{bmatrix}^T$, we derive the following inequality:

$$\Delta V(\tilde{x}^-) + e_{obs}^T e_{obs} - \gamma x_{ref}^T x_{ref} < 0 \tag{30}$$

1. When $x_{ref} = 0$, we can conclude that:

$$\Delta V(\tilde{x}^-) < 0$$

The closed loop trajectory converges asymptotically to the origin.

2. When $x_{ref} \neq 0$ and $V(\tilde{x}(0)) = 0$, we obtain:

$$\sum_{k=0}^{\infty} e_{obs}^T e_{obs} < \gamma \sum_{k=0}^{\infty} x_{ref}^T x_{ref}$$

The $\mathcal{L}_2$-norm of the estimation error is bounded.■

Remark 1. The inequalities (26) are LMI conditions for a given scalar ϵ. A numerical search for ϵ is carried out in a prescribed interval.

4. SIMULATION RESULTS

In this section, we will validate first the robust PI control designed in Section 3.B and then the robust observer-based control presented in Section 3.C. All the LMI conditions are solved with the Yalmip toolbox. To carry out the simulations, the nominal parameters in Table 1 are used. A second-degree polynomial is applied for the human torque approximation (16). The mass of users varies between 80kg and 120kg and the viscous friction coefficient changes between 40N.m.s and 60N.m.s.

Solving the LMI conditions in Theorem 1, the following control gains are obtained:

$$LM^{-1} = \begin{bmatrix} -315.25 & 180.95 & 69.36 & -38.57 \\ 180.95 & -315.28 & -38.57 & 69.36 \end{bmatrix}$$

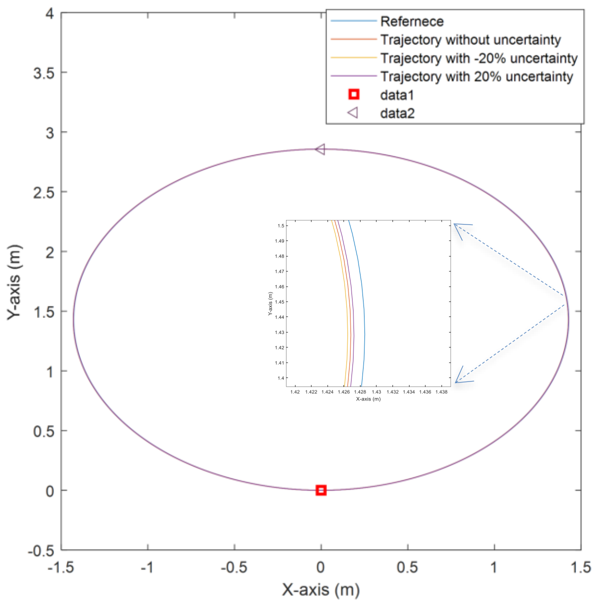

Figure 2: Obtained trajectories with the proposed robust PI-like controller

As shown in Fig. 2, the proposed PI-like controller is able to track well the given reference trajectory even the uncertainties on the mass of users and the viscous friction coefficient are present.

Solving the LMI conditions in Theorem 2, the following observer gains are obtained:

$$G^{-1}W = \begin{bmatrix} 20.58 & -11.50 \\ -12.51 & 20.30 \\ 135.52 & -63.53 \\ 152.37 & -68.56 \\ -69.00 & 131.04 \\ -75.60 & 145.22 \end{bmatrix}$$

As shown in Fig. 3, the proposed observer-based controller has nearly the tracking performance as the previous PI-like controller. The wheelchair follows the given reference trajectory despite of the uncertainties on the mass of users and the viscous friction coefficient. Moreover, as shown in Fig. 4, the observer provides a satisfying estimation of human torques without using any torque sensor. These estimated signals are crucial for generating desired reference signals (Feng, et al. 2018 A). These simulation results confirm the advantage of the proposed observer-based controller.

5. CONCLUSION

A robust observer-based tracking controller was proposed for PAWs. Using a polytopic representation, the control design is formulated as a two-steps LMI optimization problems. The first step is to design PI-like control assuming human torques are measured. Using the obtained control gain, the observer gains are derived by solving the proposed LMIs such that the stability of the closed-loop system and the estimation performance are guaranteed. Finally, the simulation results show that the observer-based controller has almost the same tracking performance as the PI-like controller. Moreover, we can successfully estimate human torques without using torque sensor. For future works, the proposed observer-based controller will be implemented to the PAW prototype designed by AutoNomad Mobility.

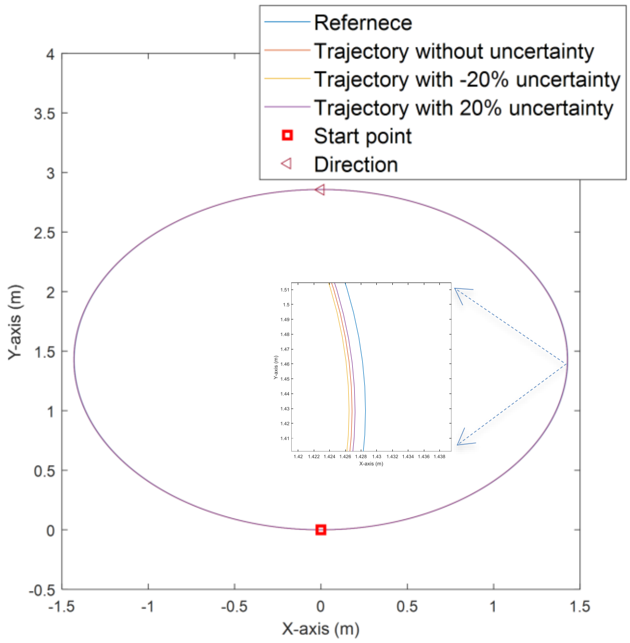

Figure 3: Simulation results with the proposed observer-based controller

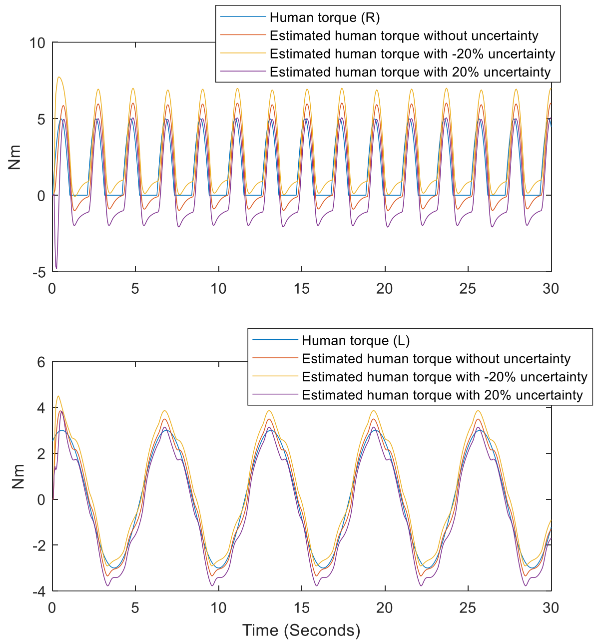

Figure 4: Obtained estimation of human torques

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